Variational phase-field modeling of cohesive fracture with flexibly tunable strength surface

Francesco Vicentini*[1], Jonas Heinzmann[1], Pietro Carrara[1], and Laura De Lorenzis†[1]

[1]ETH Zurich – Switzerland

Abstract

Variational phase-field models for brittle fracture (1) are powerful computational tools for studying Griffith-type crack **propagation** under complex three-dimensional and multi-axial loading scenarios. However, they struggle to accurately capture fracture **nucleation**, i.e., the onset of cracks in quasi-brittle materials. While effective for tensile-driven fractures (mode I) (2), they fail under multiaxial loading due to the lack of flexibility in prescribing a **material-specific strength surface** (3).
Traditional energy decomposition approaches often lead to questionable residual stresses (4), prompting non-variational modifications (5) that sacrifice the physical, mathematical, and numerical advantages of an energy minimization framework. This limitation stems from the fact that classical phase-field models merely regularize the sharp Griffith fracture model (6), which lacks a nucleation concept, unlike sharp cohesive fracture models.

To overcome this, we propose a **variational** phase-field model that approximates **cohesive fracture** allowing the inclusion of an **arbitrary strength surface** as a material property (7). Additionally, similar to what was observed in (8), we demonstrate that this formulation enables **sharp cohesive fractures**, providing an approximation that avoids smearing of the displacement field. The model naturally incorporates a sharp **non-interpenetration** condition, thus eliminating the need for additional energy decompositions.

REFERENCES

(1) Bourdin, B., Francfort, G.A. and Marigo, J.J., 2000. Numerical experiments in revisited brittle fracture. *Journal of the Mechanics and Physics of Solids*, *48*(4), pp.797-826.

(2) Tanné, E., Li, T., Bourdin, B., Marigo, J.J. and Maurini, C., 2018. Crack nucleation in variational phase-field models of brittle fracture. *Journal of the Mechanics and Physics of Solids*, *110*, pp.80-99.

(3) Lopez-Pamies, O., Dolbow, J.E., Francfort, G.A. and Larsen, C.J., 2025. Classical variational phase-field models cannot predict fracture nucleation. *Computer Methods in Applied Mechanics and Engineering*, *433*, p.117520.

(4) Vicentini, F., Zolesi, C., Carrara, P., Maurini, C. and De Lorenzis, L., 2024. On the energy decomposition in variational phase-field models for brittle fracture under multi-axial

*Speaker
†Corresponding author: ldelorenzis@ethz.ch

stress states. *International Journal of Fracture*, *247*(3), pp.291-317.

(5) Kumar, A., Bourdin, B., Francfort, G.A. and Lopez-Pamies, O., 2020. Revisiting nucleation in the phase-field approach to brittle fracture. *Journal of the Mechanics and Physics of Solids*, *142*, p.104027.

(6) Griffith, A.A., 1921. VI. The phenomena of rupture and flow in solids. *Philosophical transactions of the royal society of london. Series A, containing papers of a mathematical or physical character*, *221*(582-593), pp.163-198.

(7) Vicentini, F., Heinzmann, J., Carrara, P. and De Lorenzis L. Variational phase-field modeling of cohesive fracture with flexibly tunable strength surface. Pre-print.

(8) Alessi, R., Marigo, J.J. and Vidoli, S., 2014. Gradient damage models coupled with plasticity and nucleation of cohesive cracks. *Archive for Rational Mechanics and Analysis*, *214*, pp.575-615.