
Second-order homogenization is variational (in dimension 1)

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Abstract

When applied to one-dimensional periodic elastic lattices, the method of two-scale expansions produces an elastic energy that depends on both the strain and the gradient of strain, and therefore fits into the ‘strain gradient theory’ of Mindlin (Int. J. Sol. Struct., 1968). The energy is expressed in terms of both a scale separation parameter $\epsilon < 0$. This key property is absent from Mindlin’s theory. We show that it constrains the higher-order tractions that are applied at the boundaries of the equivalent strain-gradient continuum. The particular form of the boundary tractions makes it possible to rewrite the boundary energy in a compact and largely universal form. The class of strain-gradient models derived in this way allow second-order homogenization to be revisited as the minimization of a positive energy functional.

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