
Scaling laws in the elasticity of additively manufactured woodpile lattice structures

Atul Bhaskar*¹

¹Faculty of Engineering Physical Sciences, University of Southampton, SO17 1BJ – United Kingdom

Abstract

Porous and cellular solids are known to exhibit modulus-porosity relationships resulting from the dominant mechanism of deformation leading to their apparent elasticity (1). The well-known relationship of cubic and quadratic dependence of the apparent modulus of elasticity on volume fraction, for 2D and 3D lattices and open cell foams, respectively, may not be valid for certain lattice geometries even when the deformation is bending dominated. Here we summarise and report some novel scaling results for bending dominated deformation in woodpile lattices that are not in line with these. We present simple mechanics of woodpile structures where the deformation mechanism is neither bending nor stretch, yet when scaling arguments can be effectively applied. Here we present the mechanics of additively manufactured woodpile lattices to assess their apparent elasticity. Analytical, computational and experimental approaches are employed.

Consider a woodpile lattice with alternate layers stacked in a staggered arrangement (2). When such a lattice is compressed in the stacking direction, the individual members are in flexure. For lattice spacing much larger than the characteristic cross-sectional dimensions, the scenario can be approximated as one of beam bending, leading to a surprisingly high exponent in the fifth power law with respect to volume fraction. This result is validated against numerical and physical experiments on additively manufactured samples, showing excellent agreement with the fifth power law discovered using a simple scaling argument, when the spacing is much larger than the diameter. For dense solids, the departure from the power law scaling is observed in both experiments on 3D printed samples as well as finite element numerical results. This departure is further quantitatively accounted for by including the role of shear correction when the spacing is not very large compared to the diameter of the struts. Following the analysis, computation, and experimentation on woodpile lattices with staggered arrangement, consider the same lattices when the alternate layers are aligned (3). Under compressive loading in the stacking direction for such a lattice, each of the elastic cylinders are diametrically compressed and the deformation mechanism is neither bending nor stretch but one requiring rather complex elasticity solutions. Here we take an alternative approach of using scaling arguments that lead to a quadratic power law when the dominant mechanism of deformation is diametrical compression. Excellent agreement with numerical results as well as laboratory experiments on additively manufactured lattices is reported. Deviations from the power law for high density lattices is explained. The influence of response due to neighbouring lattice sites is quantitatively assessed and shown to be small. Variational approximations based on an exponential ansatz for spatial decay match extremely well with numerical computations.

*Speaker

Finally, results for the apparent elasticity of a hierarchically porous woodpile lattice with hollow cylindrical tubes are presented (4). The result is not a simple power law, yet an analytically tractable elegant expression in terms of the volume fraction and the ratio of thickness to the radius of hollow struts. The influence of deformation at neighbouring sites is quantitatively examined. Simple analyses, computations and laboratory experiments on additively manufactured samples using bespoke nozzles are reconciled and found to be in excellent agreement.

References

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