Tanh outperforms power-law for modeling experimental hardening of UHS steels

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Abstract

The accurate modeling of tensile behavior in ultra-high-strength steels (UHSS) presents a significat challenge due to the inherent limitations of conventional power-law-based models in capturing the transition and strain-hardening behavior across diverse deformation regimes. Traditional models, such as the power-law and its derivatives, assume a constant strain-hardening exponent and linear logarithmic relationships between true stress and true strain. These assumptions fail to account for the nonlinearities observed in experimental stress-strain curves in logrithmic space, particularly in the transition and strain-hardening regions. Consequently, conventional models often yield significant discrepancies when applied to materials with limited ductility, such as UHSS grade S960MC.This study proposes a novel hyperbolic tangent (tanh)-based hardening model to overcome these limitations. The model is mathematically expressed as: σ***=atanh(***α***)+btanh(***β***)***, where σ represents the true stress, is the true strain, a corresponds to the stress contribution in the elastic region, equal to the yield strength (Y), b corresponds to the stress increment due to hardening, defined as UTS–Y, α and β are scaling parameters controlling the strain's influence on stress response in the elastic and plastic regions, respectively.The first term in the model captures the elastic behavior of the material, transitioning smoothly to the yield strength, while the second term governs the nonlinear hardening response in the plastic deformation region. A key advantage of this formulation is its ability to provide a smooth and continuous transition between elastic and plastic behavior, avoiding the unphysical stress predictions often encountered in power-law models. Additionally, the derivative of the stress-strain curve at zero strain corresponds to the Young's modulus (E), satisfying the boundary conditions required for accurate material modeling.The model parameters were fitted to experimental tensile data of S960MC UHSS using a non-linear least squares approach implemented via the Levenberg-Marquardt (LM) algorithm. This iterative optimization minimized residual errors, resulting in an accurate representation of the stress-strain behavior across elastic, transition, and strain-hardening regimes.The proposed tanh-based model demonstrated significant advantages over traditional power-law models. Maximum errors in the transition and hardening regions were significantly reduced. By incorporating key material properties such as E, Y and UTS directly into its formulation, the tanh model offers both physical relevance and predictive accuracy.

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