Modeling ribbons/ strips as a Cosserat rod

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Abstract

Strip (or Ribbon)-like structures refer to elongated objects that are flat and have a narrow width compared to their length. Such structures are used in a wide range of fields encompassing various applications such as in building blocks for fabricating nanodevices, nanoelectronics and nanostructured materials, sensors and actuators, photonic waveguides, light-emitting devices, etc. For an effective modeling of strips/ ribbons as a Cosserat rod, an accurate knowledge of the strip's constitutive relations in terms of the rod's strain is essential. This study presents a computational approach to obtain nonlinearly elastic constitutive relations of strip/ribbon-like structures modeled as a special Cosserat rod. Starting with the description of strips as a general Cosserat plate, the strip is first subjected to a strain field which is uniform along its length. The Helical Cauchy-Born rule is used to impose this uniform strain field which deforms the strip into a six-parameter family of helical configurations-the six parameters here correspond to the six strain measures of rod theory. Two vector variables are introduced to model the position of the deformed centerline of the strip's cross-section and to model orientation of thickness lines along the strip's width. The minimization of the strip's plate energy together with the aforementioned uniformity in strain field reduces the partial differential equations of plate theory from the entire mid-plane of the strip to just a system of nonlinear ordinary differential equations along the strip's width line for the above mentioned two vector variables. A nonlinear finite element formulation is further presented to solve the above mentioned set of ordinary differential equations. This, in turn, yields the strip's stored energy per unit length as well as the induced internal force, moment and stiffnesses of the strip for every prescribed set of six strain measures of rod theory. The developed model is used to study the uniform bending, twisting, and shearing of a strip. For the case of uniform twisting and shearing, the strip is also seen to buckle along its width into a more complex configuration, which are accurately captured by the developed model. This model involves the system of non-linear ordinary differential equation and therefore found to be computationally efficient and accurate when compared with the model of Arora et al.(2), as their scheme requires solving the partial differential equation. Further, the result from developed computational scheme is compared with the analytical scheme of Audoly and Neukirch (3) wherein they made several assumptions to come up with their analytical results. Finally, the developed model accurately obtained non-linear constitutive relation of strip when modeled as rod.
(1) R. Kumar, V. Agarwal, and A. Kumar, "A computational approach to obtain nonlinearly elastic constitutive relations of strips modeled as a special cosserat rod," Computer Methods in Applied Mechanics and Engineering, vol. 418, p. 116553, 2024.

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(2) A. Arora, A. Kumar, and P. Steinmann, "A computational approach to obtain nonlinearly elastic constitutive relations of special cosserat rods," Computer Methods in Applied Mechanics and Engineering, vol. 350, pp. 295–314, 2019.
(3) B. Audoly and S. Neukirch, "A one-dimensional model for elastic ribbons: a little stretching makes a big difference," Journal of the Mechanics and Physics of Solids, vol. 153, p. 104457, 2021.