The elastic field in the vicinity of a cylindrical defect with nanosized irregularities under plane stress

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Abstract

Stress concentrations caused by various structural and material defects are one of the main reasons why devices fail. Most modern materials used for structural and functional purposes are elastically heterogeneous. Typical examples of heterogeneous materials include fiber composites with metal, polymer, and ceramic matrix, which can have voids, holes, or specialized geometric shapes. Carbon fiber, known for its high strength and rigidity, is commonly used in the aviation industry. Other examples include aluminum alloys with pores and steel alloys that contain holes shapes. At the nanoscale, we find polymer nanocomposites with nanotubes, where the nanotubes are embedded within a polymer matrix to enhance strength, stiffness, and thermal conductivity. Additionally, graphene with defects and nanowires that contain voids are also notable examples of these materials. In these materials, fibers with different scales are distributed in different ways. The fibers range in diameter from hundreds of microns to several nanometers. The stability, strength, and physicochemical properties of these materials depend on the stress-strain state of the near-surface and boundary layers. The classical theory of elasticity cannot accurately assess real deformations and stresses when dealing with micro- and nanoscale inhomogeneities.
Due to the rapid development of nanotechnology, it is necessary to study the mechanical behaviors of nanoscale structures used in manufacturing electromechanical devices such as vibration shock sensors, biosensors, accelerometers, resonators, etc. Modeling their behavior is made possible by the tools provided by continuum mechanics. Without loss of generality, we can say that classical methods of mechanics in conjunction with models of surface elasticity (1) make it possible to simulate the behavior of nanostructures of a wide variety of nature. In recent years, the theory of elasticity has been widely used, taking surface and interfacial stresses into account (2, 3). The appearance of surface stresses in solids is associated with different conditions of equilibrium for atoms inside the body and near its surface. In a way, there is free surface energy, which leads to the appearance of surface stresses introduced by Gibbs (4).

In this work, we present the problem of a nearly circular defect in a nanoplate in a case of plane stress (5), allowing for the surface elasticity and residual surface stress by the Gurtin - Murdoch model (2, 3). The problem on a plane stress of a plate in the presence of the surface stresses differs essentially from the corresponding problem on a plane strain of a

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body, as the elastic parameters of the plate depend on the elastic parameters of the surface and plate thickness. The boundary conditions are derived according to the corresponding generalized Laplace – Young law. With the help of Goursat – Kolosov complex potentials and Muskhelishvili representations, the solution of the problem is reduced to the singular integro-differential equations. The algorithm of solving the integral equations is constructed in the form of a power series, as in (6). Based on the explicit forms of the analytical solution, we present numerical results for the stress field near the boundary of the nanopore given by the cosine function. The effect of plate thickness on the stress field at the surface of the pore and the role of surface tension at this surface is shown.

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