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# Optimizing Biaxial Test Protocols for Hyperelastic Material Characterization in Lode Invariant Space

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## Abstract

The choice of an invariant basis for the strain energy density function  $W$  for an isotropic hyperelastic material is as crucial as getting an appropriate form for the function. The conventional approach of employing the principal invariants of the Cauchy-Green tensor, though mathematically simple, does not provide direct insight into the mode-dependent mechanical response of the material. The Lode invariants ( $K_1$ ,  $K_2$ ,  $K_3$ ) of the Hencky strain tensor quantify the magnitude of dilation, the magnitude of distortion and the mode of distortion respectively. Consequently, it yields an orthogonal tensor basis for the Cauchy stress tensor that does not propagate the measurement error during biaxial experiments. Further, these invariants are better suited for analytical inverse procedures for the determination of  $W$  from mechanical data. To that end, obtaining mechanical data that sufficiently span the  $K_2$ - $K_3$  space, in addition to fixed-mode tests such as uniaxial and equi-biaxial tests, is essential to better characterize the material.

This work focuses on designing the test protocols in such a way that it spans most of the  $K_2$ - $K_3$  space with as few planar biaxial tests as possible. Intermediate  $K_3$  values are achieved by performing unequal-biaxial tests. Digital Image Correlation(DIC) method is utilized to calculate the values of  $K_2$  using data obtained from experiments on hyperelastic silicon rubber. These data points are then used to construct a grid in  $K_2$ - $K_3$  space along with their corresponding normal stresses. We demonstrate that the derivatives of the energy density function with respect to Lode invariants are much well behaved in the neighborhood of the origin compared to the same with principal invariants. By systematically exploring the Lode invariants and their role in constitutive modeling, this work offers valuable insights into material behavior under diverse and complex deformation states.

**Keywords:** *Energy Density Function; Lode Invariants; Hyperelastic Rubber; Isotropy; Biaxial Test; Digital Image Correlation*

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