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# On the localization of a new phase domain in the vicinity of a crack

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## Abstract

*Summary.* Evolution of new phase domains induced by phase transformations of the martensitic type in the vicinity of a hole as a stress concentrator and a crack is considered with the use of numerical simulations. The propagation of the interface is described by a kinetic equation that relates the velocity of the interface with a configurational force equal to the jump of the normal component of the Eshelby stress tensor. Various scenarios for the evolution of the new phase domains are demonstrated. At first, it is studied how the elliptic hole can induce interface propagation even at external strain, which is insufficient for the existence of a new phase in the absence of the stress concentrator. It is examined how the elasticity parameters of the phases and the transformation strain affect the distribution of the configurational force along the interface as well as the localization of the new phase domain. Then the interconnections between a new phase domain growth and kinetics of the crack growth are considered.

A coupled problem of the evolution of a new phase domain as a result of stress-induced phase transitions in an elastic body with a crack is considered. An example of such transitions are martensitic transformations, which are accompanied by transformation strain as a result of changes in the shape and size of the crystal lattice. A consequence of martensitic transformations are shape memory effects, which are used in the design of structural elements that react non-trivially to external thermomechanical actions (1,2). The transformation of austenite into martensite is associated with the formation of a process zone (a zone of material with changed properties) in the vicinity of the crack tip - an additional source of energy dissipation during crack growth, which subsequently continues to propagate along with the crack. This leads to the need to take into account the interconnections between crack growth and the new phase domain as a process zone.

There are various approaches to analysing the interconnections between the stress-strain state and phase transitions. Some models imply the introduction of additional parameters characterizing the average structural features of the material, for which special constitutive relations are formulated. A limiting phase transformation surface in stress space can be defined similarly to yield surfaces in plasticity with the use of various criteria and correction factors (3–5).

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Other models assume the explicit introduction of interfaces and describe the evolution of new phase domains. Phase field approach and sharp-interface approach can be distinguished among the models. Phase field approach implies considering the interface as a transition layer of finite thickness where material properties vary smoothly. The changes of properties are specified by introducing order parameters, the evolution of which is described by additional Ginzburg–Landau-type equations (6–8).

In the present work a sharp-interface approach is implemented in which the thickness of the interface is neglected. The constitutive equations are written in each phase separately and additional kinetic equation or equilibrium condition is formulated for the interface. The sharp interface propagation is described within the framework of the mechanics of configurational forces (9–11). A kinetic equation relating the velocity of the interface with a configurational force equal to the jump in the normal component of the Eshelby stress tensor is used (12,13).

A numerical procedure for simulating the evolution of the new phase domain in an elastic solid is developed and verified. The stress-strain state is determined using the finite element method. The interface is considered as a surface, which passes along the edges of the mesh elements, and the remeshing of the geometry takes place on each time step.

First, various scenarios for the evolution of the new phase domain in a homogeneous body under external strain are demonstrated. It is shown that the new phase domain can itself act as a stress concentrator that promotes further phase transformation (14). Then the variety of the interface behaviours in dependence on the distance between the interface and the stress concentrator, and the form of the concentrator is demonstrated based on considerations of the configurational force distribution along the interface (15). It is examined how the elliptical hole can induce interface propagation even at external strain, which is insufficient for the existence of a new phase in the absence of the stress concentrator. After that, it is studied how the shape and degree of localization of the new phase domain in the vicinity of the tip of the elliptical hole, as the prototype of the process zone in the vicinity of a crack tip, depend on the material parameters of the phases (see also (16)). It is also discussed how the new phase growth causes stress relaxation in a body, based on examinations of stress distribution.

Finally, developed algorithms are used both for studying of interface propagation in the vicinity of a static crack and investigating the joint propagation of the crack and new phase domain. Crack kinetics is determined by the Paris law. The influence of material parameters, external strain, and the distance between the crack and the interface on the stress intensity factor and crack growth are considered.

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