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# ON MODELING COUPLED THERMOELASTICITY WITH GRADIENT DAMAGE UNDER DYNAMICS CONDITIONS

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## Abstract

Local damage models generally exhibit pathological sensibility to mesh size or orientation. A common way to improve such damage models consists in incorporating gradient-damage terms which involve a length scale. The variational setting of such models allows to link them to fracture modeling (3). Variational gradient damage models have been also extended in various directions among which those allowing to account for thermoelastic effects (generally in an uncoupled form) or for dynamics conditions (2) (4). In the footstep of these works, the present study aims at developing a variational framework for gradient damage combined with a fully coupled thermoelasticity under dynamics conditions. To this end, we rely on Biot's Lagrangian formalism of dynamics equations in presence of dissipative phenomena Biot (1) (see also (6)). This is similar to the introduction of a Rayleigh-like dissipation function in the classical Lagrange equations. Combined with global normality rule, the procedure yields a variational inequality for the dynamics gradient damage model with thermal effects. Adopting a time discretization allows to establish an incremental variational setting involving a three fields (displacement, temperature, damage) functional (except for the dynamics contribution, this has some similarities with (7),(8)). The numerical resolution of the model is performed in a staggered manner by first determining the temperature field through the solution of the heat equation, then the displacement and velocity field by means of a Newmark integration scheme and finally the damage field by a minimization of functional at the current time step with an account for the damage irreversibility condition. This is done until convergence.

For illustration purpose, we had performed various simulations of dynamic fracture propagation in a thermoelastic-damageable materials. For instance, a plate submitted to a thermal shock together with a mechanical loading (see for instance (5), also for materials properties) has been considered. The numerical results demonstrate the capabilities of the model to predict crack branching in such context. A full discussion of the temperature field, stress and displacement field, as well as the evolution of the different energies is performed.

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