The late gothic ribbed vaults of San Miguel church in Segovia: from geometric and proportional analysis to structural assessment

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Abstract

Introduction and description of case study

The present work aims to study the Church of San Miguel in Segovia (Spain), focusing on the late-Gothic ribbed vault, from a geometric and a structural point of view.

The construction of the current church began in 1532, after the demolition of a previous Romanesque church located in Plaça Mayor. The project for the reconstruction was assigned to Rodrigo Gil de Hontañón (1500-1577), one of the most prominent Spanish architects of 16th century. However, at his death the church was not completed and its construction was continued by Diego Matienzo.

The layout of the church features a central nave - composed of five bays - a transept and lateral chapels. Each bay is covered by a domical ribbed vault, the design of which is described in detail in the treatise. The ribs are of four types: cross arches *(cruceros)*, located along the diagonals; formerets (*formeros*), that are the longitudinal ones; transverse arches (*perpianos*), that are the transversal ones, that divide one bay from another; tercelets (*tiercerons*), located between the formerets or the transverse arches and the cross arches. Finally, the shape of the vault is characterized by the *ligadura*, which are ribbed elements that connect the *tiercerons* to the cross ribs, the formerets or the transverse arches, creating a floral pattern.

Identification of the ideal geometry of the vaults and comparison with the real one and with Rodrigo Gil's rules

Rodrigo Gil, who designed the church, was also the author of a Treatise of Architecture

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(1544-1554) which provides rules for the different aspects of the design of a late gothic church. Therefore the first aim of this study is to compare the real vault's geometry with the prescriptions of the treatise. The real geometry, has been obtained from the point cloud of the church which resulted from a previous laser scanner and photogrammetric survey.

The analysis started considering the central vault, at the intersection between the nave and the transept, which has an almost squared plan. From a first analysis of the plan and of some significant sections made in correspondence of the cross ribs and longitudinally in the main nave, it appears evident that the main vault of San Miguel church belongs to the type of the late gothic "*bóvedas baídas*", which are typical of the 16th century churches built in Spain. These vaults are dome-shaped. According to the ideal shape, the central part of the vault (bounded by the so called *tiercerons*) corresponds to a sphere, the cross ribs are perfect semicircles (at least in the ideal geometry), and each *tierceron* is a circular segment (since they result from the cutting of a sphere). The peripheric parts, which are the lunettes situated between the *tiercerons* and the transversal ribs, are instead formed by ruled surfaces, which serve to connect two pointed arches: one is formed by the two *tiercerons* and the other by the transversal arches or by the tercelets, whose crowns are lower than that of the *tiercerons*. Since it is not possible to find information on the shape of transverse and longitudinal arches (formerets), their geometry was evaluated starting from the point cloud and considering the radius of the two symmetric segments of the pointed arch as a fraction of the span.

These ideal shapes have been compared with the real ones of San Miguel's central vault, obtained from sections of the point cloud. The real cross ribs of this vault with squared plan are perfectly semicircular, although one is slightly deformed because of a settlement of one of the pillars that is clearly detectable and measurable from the point cloud. From the comparison between the spherical ideal surface and the real sections of the vault, it emerges that, differently from the cross ribs, the real profile of the web is a bit lowered, with a maximum error of around 10 cm. However, this is more probably due to the adjustment of the vault during construction or to later deformation than to a different design shape.

After having identified the geometric shapes forming the vault, the focus has been on the analysis of the geometric proportions of the structure. This analysis started considering the plan, since gothic master builders used the plan as reference, and compares the proportional system of San Miguel's vaults with the rules contained in the treatise by Rodrigo Gil de Hontañón. Of course, all proportions are referred to a module, a unit of measurement. According to the prescription of the treatise, the width of a nave bay is 40 units. Starting from the reference measure that sets the width (and also the length for the squared vault) to 40 units, the position of the keystones has been determined using multiples of the reference unit. For example, the keystones that form the edge of the central square are 8 units away from the perimeter of the vault. The direction in plan of the *tiercerons* was derived accordingly. Following this method, all the proportions needed to define the ideal plan were derived. By superimposing the ideal geometry thus obtained on the real plan, an almost perfect overlap can be observed.

Then the proportions of the rib sections has been analysed. Rodrigo Gil's treatise provides the rule to determine the height of the different types of ribs as a fraction of the nave's span. He also sets an analogy with the human body: each type of rib corresponds to a finger. Again, the height of the different ribs has been calculated according to Rodrigo's rule (transverse rib are 1/20 of the span, cross ribs 1/24, *tiercerons* 1/28) and then compared with sections of the real ribs obtained from the point cloud: the difference is no more than 3 cm. The treatise also provides rules to determine the height of the so called *tas-de-charge* (that is where the ashlars of the ribs start to have no more horizontal but radial joints) and of the infill at the extrados; the measure obtained according to the rules have been compared with the real one showing again no significant differences.

Subsequently, the analysis focused on the other vaults of the church, which are of rectangular plan. For these vaults, in the real geometry, the surface of the web, also in the central part,

is more distant to the ideal sphere. Indeed, the parts between the ribs are closer to ruled surfaces with just a small curvature. However, the cross ribs are almost semicircular also in vaults with rectangular plan.

The geometric model

On the basis of the results of the previously described study on the proportion of the elements that are part of the vault system, a geometric model was built. The starting point was the plan, from which the diameter of the star-shaped spherical surface contained within the *tiercerons* could be determined. Then, the lunetted parts between the *tiercerons* and the transversal arches were modelled with ruled surfaces, extruded in the direction of the joints between the voussoirs. All ribs were modelled as extrusions of the previously discussed sections on the profiles.

To complete the geometric model, it was relevant to divide the ribs in a finite number of voussoirs. This procedure was applied only for the ribs, without discretizing the web, by tracing the end surfaces of each voussoir using the photogrammetric survey of the plan. The procedure consisted in creating a sheaf of planes originating from the centre of the vault and ending in the boundaries of each voussoir, used to trim the rib in a finite number of blocks.

The final geometric model thus consists of the spherical star-shaped web and the four lunettes, both 12 cm thick, and a set of discretized ribs. A verification of the accuracy of the three-dimensional model was then conducted, comparing the intrados of the spherical surface and the lunettes with the point cloud of the survey.

Structural analysis of the main vault

The geometric model was firstly employed to assess the equilibrium and the safety of the vaulted system, by using the Membrane Equilibrium Analysis, developed under the Heyman's theory of Limit Analysis. Within this theory, it is possible to find one surface within the web of the vault, being in equilibrium under dead loads in a compressive state. Through analytical solution it is possible to find parallel and meridian stresses in each point of the surface and, therefore, the dead load on each voussoir of the ribs. In the case of the *tierceron* both stresses transmitted from the spherical surface and the lunette ruled surface have been considered and summed, to find the vertical components. Then, equilibrium assessment is done by computing the thrust line of the ribs.
After this analysis, a Finite Element Model (FEM) is used to analyse the effects of the movements of the supports. The blocks are modelled considering a sufficiently stiff materials, while interfaces are modelled considering no-tension contact, in accordance with Heyman's theory. The results in terms of crack openings are then compared to the real crack pattern, in order to verify the possible developed mechanism.