A well-posed homogenized strain-gradient model for elastic waves in periodic media.

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Abstract

This work focuses on low-to-medium frequency linear elastic waves in unbounded periodic media. In this context, we show
how to obtain a well-posed strain-gradient model using the two-scale asymptotic homogenization method, pushed to the
second-order.

We combine (i) classical formal asymptotic expansions in terms of the periodicity length-to-wavelength
ratio (1) and (ii) original reciprocity identities between the so-called cell problems at various orders to obtain new
relations between the higher-order homogenized tensors entering the model (2). The latter results allow to highlight the
symmetry properties of those tensors and reduce the overall cost of their computation for a given periodicity cell.

An original "Boussinesq trick" is then proposed, which allows to ensure the positivity and (respectively) coercivity and
ellipticity of the stiffness and inertial bilinear forms featured in the obtained strain-gradient wave equation. Those
properties, in turn, are used to establish the well-posedness of initial-value problems in the free space featuring that
equation.

Finally, numerical simulations show the expected second-order asymptotic accuracy of the model and its ability
to reproduce key features of the wave propagation, notably higher-order anisotropic propagation that would not be

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captured by a classical, leading-order homogenized model (3) and demonstrate the practical advantages gained from the
reciprocity identities.

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