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# Mean-field micromechanical incremental approaches for brittle damage in particulate microstructures under monotonic loading

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## Abstract

In this work, two mean-field micromechanical approaches are proposed to determine the effective behaviour of a particulate microstructure where damage occurs in matrix phase. The local behaviour of the phases is derived in the framework of generalised standard materials proposed by P. Germain et al. (1983) and the effective behaviour is obtained by defining an incremental potential in the spirit of N. Lahellec and P. Suquet (2007), made up of two potentials: the free energy and the dissipation, given in this work by E. Lorentz and S. Andrieux (2003).

In the first incremental approach, presented in V. Gauthier et al. (2023), the variational formulation is developed by considering in the damaged phase two sub-phases characterised by two damage states at the beginning of each time step. This approach leads to the definition of an elastic nonlinear problem at each time increment but, due to the damage evolution, the corresponding nonlinear potential is non convex. Next, this elastic nonlinear problem is regularised by introducing a linear comparison composite whose features are optimised by taking inspiration from the Full Optimised Second Order approach of P. Ponte-Castañeda (2016). However, the reduction of the number of degrees of freedom of the problem is realised by considering an isotropic linear comparison medium.

The second incremental approach uses a linear comparison medium made by brittle elastic phases, by considering in each time step a quadratic incremental potential depending on both the strain and the damage fields. The inner and outer stationary conditions lead to a system of equations that characterises the effective behaviour and gives fields statistics. Moreover, these equations also provide an uniform damage field in the damaged phase, in each time step. The value of the damage depends on the average of the relaxed stored elastic energy.

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The abilities of the proposed approaches are assessed through a particular case. The composite is made up of two elastic phases : the matrix and the inclusions. The inclusions are subjected to a monotonous increasing swelling while the boundary of the RVE is stress free. For this problem, the resulting solutions are given by closed-form expressions. These analytical developments are then used to obtain the effective response of the composite whose dependency on physical parameters (volume fraction of inclusions, toughness of the matrix, elastic contrast ...) is then analysed and compared with the results of full fields calculations.

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