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# Energetically exhaustive homothetic tests of hyperelastic isotropic materials: analytic tools and experimental characterization

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## Abstract

The mechanical characterization of hyperelastic, homogeneous, and isotropic materials is typically carried out by fitting a stress component or resultant of one or more states of stress. Uniaxial tension/compression, biaxial, torsion, and shear tests are some of the most common experimental methods for obtaining constitutive parameters. Once the material law is assumed (*Ariano, R. (1925)*, *Truesdell, C. (1956)*, and *Signorini, A. (1959)*), the target of the fitting procedure is the determination of the material constitutive parameters. Yet, it is possible to directly track the 2D or 3D behaviour of the derivatives of the energy function (DEF) of incompressible or compressible materials, respectively.

Depending on the choice of the deformation invariants and the experiment, three families of homothetic tests can be recognized (*Falope et al. (2024)*): energetically exhaustive, partially exhaustive, and non-exhaustive tests. A test is said energetically exhaustive if its matrix form is equivalent to a determined system of equations in the unknowns DEF. The strength of the unequal-biaxial test is discussed, showing that it is able alone to directly characterize the form of the DEF. Experiments of this type are presented and design tools for these tests are proposed. The outcomes of the unequal-biaxial tests on three different rubber-like materials bring to light two unexpected results. First, two empirical (*E*-)inequalities do not describe the behaviour of two tested materials, while the *Backer-Ericksen* inequalities (1954) are fulfilled. Second, new alternative forms of the BE-inequalities are presented together with a new *hierarchical empirical inequality*.

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