Nonspherical oscillations of an encapsulated magnetic microbubble

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Abstract

Encapsulated microbubbles is used in biomedical applications, serving as drug delivery carriers and contrast agents for ultrasound imaging. Recent advancements have focused on developing magnetic microbubbles that integrate diagnostic and therapeutic functions. A key phenomenon in these systems is the resonant frequency of the bubbles. While resonance should be avoided in contrast agent applications to extend the carrier time, it is exploited in drug delivery to rupture the bubble and release the drug. Therefore, understanding and modeling the resonant frequency and the stability of these bubbles is essential.
The radial dynamics of encapsulated bubble is well understood by modifying the standard Rayleigh-Plessset equation to include the encapsulation effect. To understand about non-spherical oscillation of bubbles many papers assumed non-linear radial oscillations and linear shape mode oscillations. Using these assumptions by surface elasticity theory mode shape equations is derived and surface Instability of derived shape modes is obtained (1). But, these equations were not completly valid due to consideration of linear shape modes. Recent papers have modified the non-linear shape mode interactions of gas bubble models for an encapsulated bubble and found out that non-linearity saturates the instability from spherical oscillations in case of encapsulated bubbles also (2).

There are very few works on modelling non-spherical oscillations of encapsulated bubbles under magnetic fields. This is mainly due to the fact that surface magneto elasticity is not well developed to apply to bubble surfaces. Also, due to absence of magnetic monopoles purely radial magnetic field is impossible and it is at least axisymmetric and forces on the bubble surface have both radial and tangential components. This does not allow the use of potential flow assumption and restricts usage of energy methods to obtain mode shape equations.

In this work to model non-spherical oscillations of encapsulated magnetic bubbles, instead of interface theory a simple membrane theory for Finite deformation of magneto-elastic membranes developed by Steigmann and Barham (3) is used. The membrane theory is approximaing the three dimensional elasticity equation to two dimension by leading order approximation along thickness . The assumption involved in magneto-elasticity is that the membrane is a poymer coated with weakly magnetic particles. This allows to neglect induced magnetic fields and magnetic forcing is only througn applied magnetic field and field gradients created by coils symmetrically placed above and below the bubble. The bubble's deformed shape is expressed using radial and tangential shape modes and by using orthogonality of Legendre polynomials, individual shape mode equations is derived.

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But, the derived equations encounter similar limitations as linear shape mode approximation of non-magnetic bubbles. The dynamics of bubble when nonlinear mode interaction is considered is unknown as our equations starts behaving exponentially when applied pressure reaches a critical value at given driving frequency which may or may not be actual behaviour of the bubble. However, there is a region in applied pressure amplitude-frequency plane in which our equations are valid. These regions are computationally found and it is observed that this region geneally increases with higher driving frequencies and some sudden dips in pressure amplitude is observed which may be caused due to standard as well as sub harmonic resonances. Using the approximation that toroidal vortex in the fluid is confined to a small boundary layer near bubble surface natural frequency of each modes is obtained computationally. Also, how the stability diagrams vary with various material parameters is computationally studied.

Despite its limitations, this model provides a foundational framework for analyzing the behavior of coated magnetic microbubbles Within the critical pressure amplitude and frequency ranges. This approach enables the study of their dynamics, paving the way for future research to address non-linear interactions and refine the model further.

References-

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