Mechanics of Magnetic Gel Balloons

Nadeem Shaikh*†[1] and Ganesh Tamadapu[1]

[1]Indian Institute of Technology Madras – India

Abstract

The inflation of polymeric balloons presents a fascinating challenge due to the involvement of large deformations in nonlinear materials. This complex interplay of geometric and material nonlinearities renders the governing differential equations susceptible to various types of bifurcations. These bifurcations, in turn, manifest as distinct physical instabilities, each with unique characteristics and implications.
One prominent phenomenon is limit point instability, where the balloon undergoes a sudden and uncontrollable expansion (burst) when the internal pressure reaches a critical threshold. This is characterised by a non-monotonic relationship between internal pressure and material stretch, signalling a loss of stability. Another instability, symmetry-breaking, occurs when the inflated balloon deviates from its initial mirror-symmetric shape, adopting an asymmetric configuration as the structure loses balance across a plane of symmetry.

Shape bifurcation is yet another form of instability, where the balloon transitions between distinct geometric configurations, such as shifting from a spherical to a pear-shaped (1) structure under certain conditions. Additionally, wrinkling arises due to localised compressive stresses exceeding the material's ability to resist deformation (2). This results in crumpling or folding of the membrane, a consequence of its minimal bending stiffness.

These instability-related phenomena not only illustrate the complex mechanics of polymeric balloons but also highlight their sensitivity to material properties and loading conditions, making them a compelling and interesting subject.

Interestingly, balloons made from active materials such as gels, piezoelectric polymers, or magneto-elastic composites introduce additional control parameters. The application of external fields to these materials offers an exciting avenue to investigate how these fields influence the onset and evolution of instabilities, providing deeper insights into their behaviour and potential applications.

In this work, we investigate the inflation mechanics of an initially spherical balloon made of magnetic gel, focusing on the influence of a magnetic field on various instability-related phenomena. A nonuniform magnetic field is generated by a DC current-carrying circular coil placed around the equator of the balloon, combined with the application of internal hydrostatic pressure. The analysis employs an energy formulation incorporating magneto-mechanical coupling, which accounts for the magnetisation energy and the material strain energy described by the Neo-Hookean and six-parameter Ogden material models. The inflation process is classified into two regimes based on the pressurisation duration, tp: slow

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*Speaker

†Corresponding author: am19d029@smail.iitm.ac.in

inflation and fast inflation. During slow inflation, sufficient time allows solvent molecules to migrate into or out of the gel membrane, leading to compressible inflation behaviour. In contrast, during fast inflation, the movement of solvent molecules is restricted, resulting in incompressible inflation. The impact of the applied magnetic field on the balloon's behaviour is systematically analysed within both slow and fast inflation regimes, providing insights into the complex interplay between magnetic fields and the balloon's mechanical response.

The deformation of the balloon is assumed to be axisymmetric, allowing the construction of the left Cauchy-Green deformation tensor, C. The governing differential equations for axisymmetric deformation are derived using a variational formulation, which yields a system of coupled nonlinear ordinary differential equations (ODEs). These equations, together with the specified geometric boundary conditions, define the problem as a two-point boundary value problem.

To solve this problem, the shooting method is employed in conjunction with the Nelder-Mead optimisation technique. Computational results indicate that the intensity of the magnetic field has a significant effect on the limit point instability pressure for both slow and fast inflation regimes. To further analyse the stability of the deformed axisymmetric configuration, linear perturbations are introduced using Legendre polynomials. A pronounced influence of the magnetic field on the symmetry-breaking bifurcation point and the post-bifurcation equilibrium branch is observed. Additionally, the effect of material parameters on the symmetry-breaking bifurcation point and the post-bifurcation equilibrium branch is thoroughly investigated, shedding light on their role in the system's stability and deformation behaviour.

References

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