An incremental variational approach and computational homogenization for composites with evolving damage

Ghita Ben-El-Barguia[1], Sophie Dartois*†[1], Noel Lahellec[2], and Djimedo Kondo[1]

[1]Sorbonne Université – CNRS, Institut Jean Le Rond d'Alembert, F-75005 Paris – France

[2]Aix Marseille Université – Laboratoire de Mécanique et d'Acoustique (LMA) – France

Abstract

Various brittle matrix composites experience damage mechanisms caused by the initiation and growth of microcracks under mechanical loadings. This damage results in a softening effective response, ultimately culminating in failure. The main objective of the present study is to establish a mean-field homogenization model for composites with elasto-damageable constituents, by relying on the incremental variational procedure developed so far for elasto(visco)plastic composites. To achieve this, the Effective Internal Variable (EIV) method initially proposed by Lahellec and Suquet (1) is adopted (see also (2) & (3)).
Based on the variational principle proposed by (4) and the EIV method, a time discretization of the corresponding equations is proposed and a condensed incremental potential J(x,ϵ,d), which involve the free energy and the dissipation potential for the damageable constituents, is introduced. This enables the application of Ponte-Castañeda's homogenization variational procedure for non-linear behaviors described by means of a single potential (see (5)). A linearization of the elasto-damageable behavior is then performed leading to an approximation of J by J_0, the latter being the condensed potential of a linear comparison composite (LCC) with per-phase uniform parameters. An optimization with respect to the parameters introduced in J_0 provides in the end the final estimate of the effective condensed incremental potential. The macroscopic behavior can eventually be determined using a linear homogenization scheme; Hashin-Shtrikman bounds are here chosen.

The developed incremental variational procedure is then applied to composites composed of linear elastic spherical particles isotropically distributed in a damageable matrix. Thanks to a numerical solving of the developed model, an in-depth analysis of damage and deformation fields statistics, including phase averages, second moments, and field fluctuations can additionally be carried out (available in (6)).

The model is further extended by including positive hardening concurrent with damage progression. To this aim a new dissipation potential is proposed leading to a form of J_0 in which damage threshold is affine with damage level.

Performances of the model are illustrated by comparing its predictions for various loads to Finite Element (FE) full-field simulations carried out using a regularized gradient damage model (so-called "phase-field" model, see for instance (7)). A satisfactory agreement

*Speaker

†Corresponding author: sophie.dartois@sorbonne-universite.fr

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between the theoretical model and the FE simulations in terms of macroscopic behavior is first observed. At the scale of constituents, the predictions of the model for the local behavior and stress fluctuations in the matrix are also close to the FE ones. Estimations of the local behavior in the inclusion provided by the model are yet only in a qualitative agreement with the FE ones. At last, as far as damage is concerned, the evolution of d in the model demonstrates a good agreement with the spatial average of the same quantity in FE simulations even though the model equations imply a more progressive and diffuse evolution of damage within the matrix than what the damage patterns of full-field simulations predicts.

In conclusion, a homogenization model able to predict the effective response of elasto-damageable composites was proposed and first validations (by means of Finite Element computations) were provided. Current research efforts are directed toward exploring elasto-plastic composites with evolving damage.

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