
Combining continuous discrete lattice methods and Data Driven Identification to go beyond linear elasticity

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Abstract

In the field of isotropic elasticity and brittle fracture, the use of a discrete lattice network of mechanical bonds can be very useful to overcome issues from the continuum description in solid mechanics (Sage *et al.*, 2020; Longchamp *et al.*, 2024). Thus, in the field of the so-called ‘discrete element methodology’ (DEM), a Bernoulli beam kinematic can be chosen as a powerful link between nodes in a 3D space to bring out a continuous behaviour.

The cohesive beam model needs to be identified through an inverse method, also called calibration process, that aims to find the local parameters that best approximate the macroscopic behaviour (Girardot *et al.*, 2019). These parameters have different natures. On the one hand, the density of beams, defined by the number of discrete elements and by their connectivity, refers to the lattice structure. On the other hand, the local constitutive model relative to the cohesive beams form a second set of unknowns, especially if the target continuous behaviour needs to go beyond linear elasticity. In this case, the calibration step becomes tedious and requires to solve a high-dimensional problem that involves multiple parameters.

We suggest to improve this process by employing a model-free data-driven approach for the identification of the local behaviour of the mechanical bonds. The idea of model-free computing was first introduced by (Kirchdoerfer and Ortiz, 2016) with Data-Driven Computational Mechanics (DDCM), a method in which the constitutive model traditionally employed to solve initial-boundary-value problems is replaced by a discrete database that samples the material response. DDCM inspired Data-Driven Identification (DDI), which has been developed by (Leygue *et al.*, 2018) and further refined by (Leygue, 2024).

DDI is an inverse method that aims to find the balanced stress fields from heterogeneous strain fields without postulating any constitutive model. The algorithm requires displacements fields, for instance obtained from full-field measurements, and boundary conditions on a given geometry. In the end, it provides a discrete data set, called material database, that samples the material response in an appropriate space depending on the nature of the

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material behaviour (elastic, hyperelastic, etc.). DDI can be used to compute all kinds of linear or nonlinear elastic mechanical responses without further refinements. This property differentiates model-free from model-based methods, such as Finite Element Model Updating (FEMU), where testing several constitutive laws implies high computational costs while the relevance of the *a priori* chosen one is difficult to evaluate.

The idea is to implement DDI within a DEM framework to identify a local behaviour that provides a good approximation of a macroscopic material response obtained experimentally or generated artificially (in a way that mimics a real experiment), which is also the same framework to study the continuous emergence from a discrete network. In a first stage, we build a lattice structure following recommendations to ensure an isotropic network (Girardot *et al.*, 2019). When combined with the (pseudo-)experimental nodal displacements and reaction forces, performing DDI provides a discrete material database that describes the local material behaviour.

Preliminary results have been obtained with a truss kinematic, where each link corresponds to a bar in the 3D mesh. A Finite Element (FE) simulation conducted with a nonlinear elastic constitutive model provides nodal displacements and reaction forces. Then, DDI has been performed on the truss problem in the stress-strain space defined by the Cauchy stress tensor and the infinitesimal strain tensor. These quantities are scalar and represent the material behaviour of the bars.

To compute the mechanical response of the lattice, a constitutive model can be fitted a posteriori on the local material database, either in the stress-strain space or, following the recommendation of (Costecalde *et al.*, 2024), thanks to the strain energy density. The latter is computed from the material database and provides a more accurate estimation of the parameters of the constitutive model.

Furthermore, a study of the influence of the lattice design on the material database obtained with DDI is led. We test different connectivities by linking the nodes to their n -first neighbours. Moreover, the truss is converted to a lattice beam model, which requires to define the adequate space for the description of the local behaviour.

The combination of DDI with DEM provides flexibility on the design of the lattice, since this operation is decorrelated from the identification of the local behaviour. DDI also allows to differentiate between several material behaviours with a single specimen (Valdés Alonzo *et al.*, 2022), which would make it possible to implement a complex network of bonds with varying properties.

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