Toupin-Mindlin first strain gradient elasticity for anisotropic and isotropic materials at small scales

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Abstract

Nonlocal elasticity and strain gradient elasticity theories are challenging generalized continuum theories to model crystals at small scales like the Angström-scale (see, e.g., (1,2)), where classical elasticity is not valid and leads to unphysical singularities. The theory of first strain gradient elasticity in its modern form dates back to Toupin (3) and Mindlin (4). A mathematical modeling of the elastic properties of cubic crystals with centrosymmetry at small scales by means of the Toupin-Mindlin anisotropic first strain gradient elasticity theory is presented (2). In this framework, two constitutive tensors are involved, a constitutive tensor of fourth-rank of the elastic constants and a constitutive tensor of sixth-rank of the gradient-elastic constants. The 3+11 material parameters (3 elastic and 11 gradient-elastic constants), 3 characteristic lengths and 1+6 isotropy conditions are derived. The 11 gradient-elastic constants are given in terms of the 11 gradient-elastic constants in Voigt notation. The numerical values of the obtained quantities are computed for some representative cubic materials using an interatomic potential (MEAM) (2, 5). Moreover, the isotropy conditions of strain gradient elasticity are given and discussed. A generalization of the Voigt average towards the sixth-rank constitutive tensor of the gradient-elastic constants is given to determine the 5 isotropic gradient-elastic constants (2).

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