A generalized beam model for the analysis of the mechanical behavior of beam-like metamaterials

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Abstract

Metamaterials are widely studied as they combine the practical needs for optimization and innovation with the desire to explore new frontiers in the mechanics of materials and structures. In fact, thanks to a proper design, metamaterials can exhibit exceptional characteristics overcoming those of traditional materials (1), such as low stiffness-to-weight ratio, ductile failure and exotic behaviors. Since metamaterials are often periodic structures, they can be treated as equivalent continua through the homogenization technique, which offers great advantages, including reduced computational effort and the possibility to find closed-form solutions.
Among metamaterials, there is the so-called grid beam (2), which is composed of a cellular microstructure made up of two orders of orthogonal micro-beams. For such microstructured beam, warping can be categorized into two types: micro-warping, which involves the bending of the microstructure around a plane to which the joints are constrained, and macro-warping, where the joints are free to move out of the plane. This behavior challenges traditional beam theories and has not been accurately described yet. In fact, de Saint Venant highlights its importance but neglects volume-applied loads, while Timoshenko's beam model approximately accounts it by introducing a shear factor but still assumes the hypothesis of rigid cross-sections out-of-plane. Moreover, the homogenized model for the grid beam, already existing in the literature, approximately accounts for micro-warping (2) by adopting an approach similar to that of Timoshenko but neglecting macro-warping.

To overcome these limitations, this study proposes a homogenized generalized beam model that extends classical frameworks by introducing additional strain measures to capture distortional and bidistortional effects accurately. The model is derived within the framework of the direct one-dimensional approach, while the constitutive law is determined through a homogenization procedure, i.e. the mixed homogenization approach (3), which enforces an energy equivalence between the cell, the smallest unit of the microstructured beam, and its one-dimensional counterpart of same length. The inertial properties for dynamic purposes are evaluated under the assumption of lumped masses at the cell joints. Linear static and free dynamic analysis of some grid beams, taken as case studies, are analyzed.

To highlight the effectiveness of the equivalent beam model, results obtained by the latter are compared with those derived from finite element (FE) analysis of refined models.

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