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# Some applications of a variationally consistent multi-scale model for microstructures displaying voids on boundary

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## Abstract

Materials displaying microstructures with random porous distribution are found in a myriad of ways in nature and engineering applications: foams, bones, reservoirs, architected materials with defects, etc. Minimally constrained multiscale models (Reuss-like for solid mechanics) loses the well-posedness when voids reach the boundary in a non-periodic fashion (situation which the convenient periodic boundary conditions cannot be applied). Generally, only linear boundary models (Voigt-like) can be straightforwardly applied in such cases, notwithstanding the well-known drawback of overestimation of effective stiffness. Hence, we investigate a novel multiscale mechanical formulation based on a proper formulation of a generalised minimally constrained multi-scale model that remedies this issue for general combination of solid and void parts on the boundary, as recently proposed in (1). The Method of Multiscale Virtual Power serves as the foundation for developing the proposed model, with particular focus on reformulating the gradient homogenisation formula (6). This reformulation leads to the development of not only a novel Minimally Constrained Kinematical Multiscale Model but also inspires a broader family of multiscale models suited for practical scenarios. The key ingredient is the introduction of an averaged boundary normal vector modifying the point-wise normal appearing in the non-local kinematical constraints arising in fibre networks (2). Although special attention is given to the classical first-order theory of solid mechanics at continua, the present method can be straightforwardly adapted to other applications, including strain-gradient of architected materials with defects (4), homogenisation of fibres networks (2) and fluid saturated porous media (5). Implementations aspects of such generalised kinematical constraint will be discussed in the context of FEniCSx-based opensource implementation (3). Several numerical examples demonstrating the potentialities of such method are also addressed.

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