
Simple stimuli drive complex responses in non-Euclidean active shells

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Abstract

Active materials have recently drawn the attention of the mechanical community since their response to external stimuli can be harnessed to reproduce natural phenomena. In this regard, active slender structures may display complex behaviors through mechanical instabilities. As an example, recent works have shown that polyelectrolyte hydrogel filaments actuated by a constant and uniform electric field exhibit nonreciprocal self-sustained oscillations as a consequence of flutter instability, generating a net thrust in the surrounding viscous fluid (1, 2, 3). We explore whether such nontrivial responses to simple stimuli can be achieved also in non-Euclidean active shells, i.e. shells that lack a stress-free rest state due to a geometric incompatibility caused by internal activity (4, 5). Compared to rod-like structures, active shells are found to exhibit a richer behavior at large deformations ensuing from the competition between stretching and bending energies. To capture this complexity, we adapt a nonlinear shell model of Koiter type in the context of morphoelasticity, encoding the activity by a time-evolving spontaneous spherical curvature. The mathematical model is specialized to morphoelastic elliptic plates and cylindrical shells free to move in a viscous environment, taking into account the fluid-structure interaction. We demonstrate the emergence of flutter through a linear stability analysis and predict its dependence upon the model parameters. Our analysis is confirmed by numerical simulations of the nonlinear model and by physical experiments on polyelectrolyte hydrogel shells under the action of a constant and uniform electric field. Both simulations and experiments agree in showing the achievement of oscillatory nonreciprocal motions of large amplitude. Our findings may be exploited in soft-robotics and micro-fluidics. The results we have found at the level of a single structural element open new perspectives on the possible applications of complex structures based on active materials.

References

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