A nonlinear toroidal shell model for surface morphologies and morphogenesis

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Abstract

Geometric forms are intrinsically linked to fundamental biological functions, with curvature serving as a key descriptor of local geometry that has been shown to be critical for the performance of biological systems. In addition to the classical uniform curved surfaces, when tubular structures are bent or branched, they introduce curvature in the second principal direction, leading to regions with positive or negative Gaussian curvatures, as seen in the toroidal model. However, theoretical understanding and prediction of their nonlinear instability are quite challenging. In this study, we systematically develop a nonlinear toroidal shell model based on general differential geometry and derive analytical solutions that can effectively predict critical buckling behavior. Our results analytically demonstrate that, in toroidal geometries with nonuniform curvatures, the critical instability stress is proportional to both the curvature and the stiffness, in agreement with numerical predictions. We also present a universal phase diagram of critical topographies, determined by two dimensionless parameters across a wide range of parameter space, which is further validated through experimental demonstrations.

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