A reduced-order model formulation for dispersion curve computation in nonlinear metamaterial beams

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Abstract

Elastic metamaterials have garnered significant attention in recent years for their ability to manipulate wave propagation and mitigate vibrations. These properties are typically achieved by engineering frequency band gaps through the integration of resonant structures with subwavelength dimensions into the waveguide. The design of these structures is normally ruled by dispersion laws where linear models of the resonant structures are coupled to the elastic waveguide. Although nonlinear resonators are gaining interest, they are often modeled with simplistic approaches based on single-degree-of-freedom (DOF) models. For multi-DOF nonlinear systems, capturing mode interactions is essential for accurate dispersion analysis, which significantly increases formulation complexity.

This contribution introduces a method to address this complexity by using a reduced-order model (ROM) for nonlinear multi-DOF resonators, which effectively captures mode interactions and enables efficient calculation of dispersion curves for the metamaterial. Specifically, a ROM is derived for each mode (referred to as the "master" mode) of the resonator, incorporating the effects of nonlinear restoring forces from other modes ("slave" modes). In turn, these contributions are expressed as functions of the master mode displacement and velocity using the invariant manifold approach. This allows the calculation of the ROM transfer function and coupling with a substrate to predict dispersion relations near the resonance frequency of the master mode.

The findings reveal that simplifying the original nonlinear system to a single-DOF model without accounting for interactions from other modes introduces substantial errors in the resonator response and, in turn, in the waveguide dispersion prediction, potentially leading to inaccurate metamaterial design. Conversely, the proposed ROM-based approach efficiently predicts resonator responses comparable to those of the full nonlinear model thus leading to accurate determination of dispersion curves.

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