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# On the criteria for material instability in rate-independent plasticity

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## Abstract

In solving rate-independent plasticity problems at finite deformation, there is an inherent difficulty related to the non-uniqueness of incremental solutions and the instability of a deformation path. Of particular interest is the material instability, which manifests itself in deformation patterns and microstructure formation, regardless of external loading conditions. In rate-independent crystal plasticity, this phenomenon generally occurs when the effective hardening moduli matrix is indefinite due to cross-hardening of the slip systems greater than their self-hardening. The commonly used criterion of ellipticity loss has limited application because it is based on the assumed linearized relationship between the stress and strain rates and thus excluding unloading. In contrast, the analysis here also covers the onset of a laminate deformation pattern corresponding to different unloading modes in different bands.

Material instability can also be understood in a thermodynamic sense and investigated using the Lyapunov direct method. To eliminate the influence of external loading conditions, displacements at the boundary of a deforming material element can be considered undisturbed. Then, in a quasi-static isothermal deformation process, the total internal energy increment of the thermodynamic system can be decomposed into the sum of the free energy and dissipation increments. It depends not only on the incremental displacement field, but also on the fields of other incremental variables, such as plastic multipliers or the increment of internal variables. The circumstances under which the energy increment satisfies the Lyapunov conditions for stability or instability are specified and discussed. If multiple solutions exist, which is common in rate-independent plasticity, then incremental energy minimization or quasi-minimization can be used to automatically select among them in simulations, in the potential case or non-potential case, respectively.

The effectiveness of this procedure is illustrated by the examples of large deformation of an fcc metal crystal. Along the stable solution paths, the number of simultaneously active slip systems never exceeded five. In uniaxial tension in a high-symmetry orientation, as opposed to rate-dependent models, only single or double slip activity is predicted due to the instability of the fully symmetric strain path.

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