Writhing of Plant Tendrils: Where Kirchhoff Meets Lockhart

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Abstract

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Buckling phenomena in elastic rods have recently attracted renewed interest (1,2,3). Among the well-understood singular buckling events is the writhing transition. Consider a straight rod clamped at both ends and subjected to traction T. Next, suppose internal stress is generated in the rod in a nonhomogeneous manner across its cross-section. For T=0, this generates an intrinsic curvature of the rod, *K0*, which is related to the effective residual stresses within the unloaded rod. At $T>$ 0, the writhing transition occurs at a critical internal stress, represented by a critical value *K0c*. The phenomenon is supercritical, meaning the buckling amplitude scales as (*K0–K0c*)ˆ1/2.

A spectacular realization of the writhing transition is found in plant tendrils, rod-like organs by which plants (such as cucumbers, vines, and passionflowers) attach to supports. Experiments were conducted on approximately 100 living cucumber tendrils. The time evolution of tendril curvature was monitored over three days under controlled traction T. Once a straight tendril finds a support and attaches to it, it begins generating differential intrinsic stresses. Whether these stresses cause writhing and subsequent coiling depends on the external load T. These differential stresses arise from the physiological evolution (growth) of the tendril. Conversely, tendril growth itself is stress-dependent. A standard approach for describing this relationship is implemented in Lockhart-like models (4,5). By combining insights from the Kirchhoff model of writhing and the Lockhart model of growth, we propose a simple morphoelastic model to describe the evolution of plant tendrils under axial constraints. The results align well with the experimental data, reproducing a well-identified jump associated with the writhing instability.

References:

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