On the role of body couples in linear-elastic fracture of polar dielectric continua

Behlen Lennart*[1], Daniel Wallenta[2], and Andreas Ricoeur[1]

[1]Institute of Mechanics, University of Kassel – Germany
[2]Institute of Mathematics, University of Kassel – Germany

Abstract

Linear-elastic fracture mechanics deals with unbounded stresses at the tips of sharp cracks due to external loading by introducing stress intensity factors quantifying their characteristic square root singularities. While the theory of polar fracture mechanics furthermore accounts for couple stresses, whose singularities at the crack tip are likewise described by corresponding intensity factors, body couples equally inherent in this extended theory are usually neglected. However, while explicit examples of body couples of mechanical origin are indeed scarce to nonexistent in literature, those arising in polarizable media due to electrostatic action of force have been well-known for decades. They fundamentally differ from mechanical couple stresses insofar as, in contrast to the latter, whose order of magnitude crucially depends on an internal length parameter, their distribution is uniquely determined by the macroscopic electric polarization and electric field. Depending on the electric boundary conditions of the crack faces, both of these fields may suffer square root singularities at the crack tip. As a consequence, possibly significant body couples are induced in the context of electromechanical crack problems. This work explores the impact of body couples on crack tip loading at the example of an infinite anisotropic dielectric plate with central crack subjected to remote stresses and electric fields. Upon discarding couple stresses for the sake of clarity, balances of angular and linear momentum reveal that body couples, on the one hand, directly determine the antisymmetric part of the stress tensor and, on the other hand, indirectly influence its symmetric part analogously to a body force. While the former is strictly associated with mode II loading in the problem at hand, the latter corresponds to both crack opening modes; therefore, body couples are found to generally go hand in hand with mixed mode loading.

*Speaker