A geometric nonlinear length scale in the fracture of elastic materials

Raúl Lazo-Molina*[1], Mokhtar Adda-Bedia[2], Mohit Pundir[1], and David S. Kammer†[1]

[1]Institute for Building Materials, ETH Zurich – Switzerland
[2]Laboratoire de Physique, CNRS – Université de Lyon, ENS de Lyon – France

Abstract

Soft materials like hydrogels and rubber are crucial for a wide range of applications, ranging from biomedicine to advanced engineering, mainly due to their ability to withstand large deformations without failure. These materials generally exhibit nonlinear mechanical responses, which intensify around the tip of a crack. This has motivated several experimental, theoretical, and numerical investigations, which have revealed that crack propagation instabilities, such as oscillations, are related to nonlinear length scales. For instance, the weakly nonlinear theory (1) made an important contribution to estimating the nonlinear length scale for a neo-Hookean material. This was later used to formulate a dynamic crack-tip equation of motion capable of predicting the oscillatory instability (2). On the other hand, geometric nonlinearities (GNL) have also been studied using a St. Venant-Kirchhoff material under fracture mode I (3), showing that crack propagation speeds can exceed Rayleigh's theoretical limit and reach the supershear range. Consequently, both material and geometric nonlinearities have been identified as key factors in explaining different aspects of dynamic fracture phenomena. Therefore, given the limitations of traditional linear elastic fracture mechanics (LEFM) in explaining these effects, we aim to elucidate the fundamental role of nonlinearities in the fracture process of elastic materials.

This work presents a combined numerical and theoretical investigation into the effects of geometric nonlinearities on a static crack under mode I loading. Our findings reveal that a geometric nonlinear length scale, inherent to elastic materials, can be directly computed from the stress intensity factor. Furthermore, we show that the crack tip opening displacement (CTOD) serves as a valuable tool for extracting essential nonlinear parameters to characterize fracture in elastic materials, highlighting its potential as a laboratory tool. As such, this research establishes GNL as a foundational requirement for the development of a nonlinear elastic fracture mechanics theory (NLEFM) and provides the starting point for investigating dynamic fracture instabilities.

References:

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*Speaker

†Corresponding author: dkammer@ethz.ch

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