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# Accurate frictional contact algorithms for the numerical exploration of the mechanics of fibrous assemblies

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## Abstract

The goal of this work is to explore novel algorithms to enhance the predictability of fibre assembly simulators, without sacrificing the complexity of the target scenarios. Our work relies on the super-helix curvature-based discrete model for Kirchhoff elastic rods (1), coupled with the non-smooth so-bogus (2) solver for frictional contact resolution. While previously validated at the geometrical level in both 2D and 3D configurations (3), the contact forces produced by the simulations can exhibit spurious jumps, inconsistent with smoothly sliding configurations. Starting from a detailed analysis of a simulation of the three point bending test, we show that these jumps are not specific to our model, but result from the use of low order contact detection methods. We propose new algorithms to achieve efficient and accurate high-order contact detection (4), thereby retrieving artefact-free forces, as expected from our high-order discretisation.

In the 2D case, an analytic solution exists for our specific discretisation and is simple to implement in our current framework.

There is however no general solution in 3D, where the problem becomes non convex. Instead we propose a branch-and-bound approach to find pairs of closest points between two curves. Our algorithm applies to a wider range of curves than the analytic approach used in 2D. The only requirement is to have a bound on the norm of the second derivative of the curve. Our method proves on par with low order schemes in terms of computational cost, while possessing much better scaling in terms of accuracy.

As an application in the 2D case, we investigate the mechanical response of a random stack of bidimensional flexible open rings subject to compression cycles (5).

Our approach is first validated against theoretical predictions for the snap-fit of a single ring onto a cylinder, which also serves as a calibration protocol to determine the physical parameters.

We then carefully compare simulations and experiments of random assemblies of 30 rings

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under compression-decompression cycles, and demonstrate an excellent agreement between computed and measured forces.

This allows us to rely on the numerical models to further investigate the role of the friction and the geometry of the rings on the overall behaviour of the material.

The 3D case proves richer and more complex. Our preliminary results show the wide impact the spurious jumps have on the overall force on the scenario of combing hair of various natural curvatures.

#### Reference:

- (1) F. Bertails, B. Audoly, M.-P. Cani, B. Querleux, F. Leroy, J.-L. L'évêque. Super-Helices for Predicting the Dynamics of Natural Hair, *ACM Trans. Graph.* 25 (2006).
- (2) G. Daviet, F. Bertails-Descoubes, L. Boissieux. A hybrid iterative solver for robustly capturing coulomb friction in hair dynamics, *ACM Trans. Graph.* 139 (2011).
- (3) V. Romero, M. Ly, A.-H. Rasheed, R. Charrondière, A. Lazarus, S. Neukirch, F. Bertails-Descoubes. Physical validation of simulators in Computer Graphics: A new framework dedicated to slender elastic structures and frictional contact, *ACM Trans. Graph.* 40 (2021).
- (4) O. Crespel, E. Hohnadel, T. Métivet, F. Bertails-Descoubes. Contact detection between fibres: high order makes a difference, *ACM Trans. Graph.* 43 (2024).
- (5) T. G. Sano, E. Hohnadel, T. Kawata, T. Métivet, F. Bertails-Descoubes. Randomly stacked open-cylindrical shells as a functional mechanical device, *Commun. Mater.* 4 (2023).