Dominantly Funicular Arches via Active Supports

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Abstract

Funicular structures are highly efficient by proper choice of form but sensitive to variations of load distributions, i.e., their performance depends strongly on their shape. We aim to explore the quasi-static shape adaptation of elastic arches by active supports. We investigate an incompressible, unshearable, 2D, prismatic elastica using a geometrically exact rod theory. Curved shapes are enforced by non-conform, statically indeterminate boundary conditions: the two endpoints are fixed at points A and B with a distance smaller than the length of the rod. The tangent angles alpha and beta at the two endpoints are set by actuators. The external load is a quasi-static vertical, distributed force.
Using a variational formulation, we show that the equilibrium paths of the problem can be related to the classical bifurcation diagram of Euler buckling, and specifically, the stability of an equilibrium solution is determined by the magnitude of the axial load, a Lagrange multiplier in the variational setting.

We study several optimization goals to be achieved via slowly varied tangent angles alpha and beta:

(i) maximize the critical load intensity associated with the planar stability failure of the structure. Nonetheless, maximization of the load intensity leads to shapes where the internal axial force is dominant, and internal bending plays a secondary role, i.e., we expect close-to-funicular shapes corresponding to the load. Perfect funicularity is, however, unlikely as the system is underactuated, i.e., only a limited set of shapes can be enforced by the two control parameters alpha, and beta.

(ii) minimize the deviation from perfect funicularity (i.e., vanishing internal bending) for a given load distribution.
(iii) generate paths of quasi-static transformation from an initial to a target shape without dynamic instability (e.g. snapping).

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