On Nitsche's method for higher-order boundary conditions in coupled gradient-elastic Cahn-Hilliard problems

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Abstract

In the present contribution, we consider a coupled mechano-diffusion problem in the framework of Mindlin's strain gradient elasticity and Cahn-Hilliard diffusion models. This augments the modeling capabilities by incorporating elasticity-related length scales reflecting microstructural effects at the continuum level.
The higher-order boundary value problem of the coupled gradient elastic-diffusion model is formulated in a variational form within a proper Sobolev space setting. Conforming Galerkin 2D discretizations for numerical results are obtained via an isogeometric approach and implemented in Matlab.

The method implements the so-called generalized-α method for integrating the diffusion equation in time. Moreover, the time stepping procedure is paired together with an adaptive time step size to allow for stable and accurate results even for long time simulations and greatly varying time scales which are characteristic for Cahn-Hilliard models.

One drawback of the spline based finite element method arises when dealing with higher order boundary conditions that appear for the Cahn-Hilliard diffusion equation and in the theory of gradient elasticity. A strong imposition of boundary conditions cannot be easily achieved for the general case. To deal with this problem a weak imposition is achieved by utilizing Nitsche's method which has gained popularity in recent years and shares close relation with discontinuous Galerkin methods. Its advantage compared to other penalty-based approaches for weakly imposing boundary conditions lies in its easy adaptivity and desirable properties such as resulting in linear systems that avoid ill-conditioning.
In our work, we first consider a 1D problem and verify the implementation and method by comparing the results obtained to a finite-difference method. After that, some two-dimensional examples are investigated to evaluate the performance of the proposed boundary treatment. Finally, we study an example of diffusional coarsening in solders and analyze the size effects in phase separation problems by extending the model towards the strain gradient elasticity framework.

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