Analytical and numerical quasiconvex relaxation of planar Biot-type energies

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Abstract

We derive the quasiconvex relaxation of the Biot-type energy density —U-1—ˆ2, where U denotes the right Biot stretch tensor, 1 is the identity tensor and —.— is the Frobenius norm, for planar mappings in two different scenarios. First, we consider the case where the gradient of the mapping is constrained to the positive general linear group, in which case the energy can be expressed as the squared Euclidean distance to the special orthogonal group SO(2). We then allow for planar mappings with arbitrary deformation gradient; in the context of solid mechanics, this lack of determinant constraints on the deformation gradient would not exclude local self-interpenetration of matter. We demonstrate that the two resulting relaxations do not coincide and compare the analytical findings to numerical results for different relaxation approaches, including a rank-one sequential lamination algorithm, trust-region FEM calculations of representative microstructures and physics informed neuronal networks.

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