
Tensile crack propagation in disordered materials: geometrical nonlinearities, instabilities, and toughening

Mathias Lebihain^{*†1}, Antoine Sanner², Manish Vasoya, Veronique Lazarus³, and
Djimédo Kondo⁴

¹Laboratoire Navier – École des Ponts ParisTech (ENPC), Institut Polytechnique de Paris, Université
Gustave Eiffel, CNRS – France

²Institute for Building Materials, ETH Zurich – Switzerland

³Institut des Sciences de la Mécanique et Applications Industrielles – ENSTA Paris, Institut
Polytechnique de Paris, Electricité de France - EDF, CEA Paris-Saclay, CNRS – France

⁴Institut Jean Le Rond d’Alembert, Sorbonne Université, CNRS – Institut Jean Le Rond d’Alembert,
Sorbonne Université – France

Abstract

Over millions of years, Nature has evolved tough and strong biological materials through an intricate patterning of their microstructures across multiple scales. The principles underlying these multiscale reinforcement strategies have been successfully applied to create fracture-resistant bioinspired composites. However, modern challenges require optimizing the fracture properties of engineering materials based on constraints distinct from those found in Nature, such as environmental performance, fabrication process requirements, or resource availability. Nature’s blueprints, which result from complex evolutionary processes, may not be adequate to address these new sets of constraints. This motivates the construction of a comprehensive homogenization theory for tensile fracture properties to swiftly screen the vast compositional space of hierarchical composites and design tougher materials.

Yet, the prediction of the macroscopic fracture resistance from a microstructural description of a heterogeneous material at multiple scales remains challenging. Traditional simulations methods based on the extended finite-element method or the variational (phase-field) approach of fracture often cannot describe failure processes from the scale of the heterogeneities up to the structural scale, due to limited numerical efficiency in 3D. Simulations based on the perturbation approach initiated by Rice (J. Appl. Mech, 1985) have provided valuable insights on the toughening of perfectly brittle materials by heterogeneities (Gao and Rice, J. Appl. Mech, 1989; Roux et al., Eur. J. Mech. Sol. A, 2003). They showcase unparalleled numerical efficiency, as they require only meshing of the 1D crack front to model 3D crack propagation. Yet, they build on a first-order expansion of the energy release rate with respect to the front deformation, and overlook the impact of geometrical nonlinearities arising from the front distortion. Higher-order theories are available (Leblond et al., JMPS, 2012; Lebihain et al., IJSS, 2023), but their practical implementation in a numerical code is still missing.

^{*}Speaker

[†]Corresponding author: mathias.lebihain@enpc.fr

In this work, we use variational perturbation approach of linear elastic fracture mechanics (LEFM) to model coplanar crack propagation in heterogeneous media featuring spatial variations of fracture energy G_c . Equilibrium front positions are calculated by minimizing the sum of a potential energy, estimated asymptotically through the LEFM perturbation theory, and a dissipated energy, set by the fracture energy field. This approach, recently proposed by Sanner and Pastewka (JMPS, 2022), can seamlessly integrate geometrical nonlinearities of the crack front shape, while maintaining numerical efficiency thanks to the Fast Fourier Transform (FFT). Moreover, this energetic approach provides a well-established theoretical framework (Nguyen, JMPS, 1984) to describe the instability onset during crack propagation in disordered media.

By conducting thousands of simulations of coplanar crack propagation in brittle composites, we investigate the impact of the crack size and the heterogeneity contrast on the macroscopic fracture energy. We show that the presence of heterogeneities weakens the material with respect to its reference value for cracks smaller than the heterogeneity size, and a toughening for larger crack sizes (Lebihain et al., IJSS, 2023). This transition from weakening to toughening can be attributed to geometrical nonlinearities. When the crack front perimeter is smaller than the heterogeneity size, front deformations are carried by low frequency modes only. These low frequency modes generate an overall increase of energy release rate with respect to the undeformed configuration, leading to an apparent weakening of the material. As the crack grows, the presence of heterogeneities generate high frequency modes, whose contribution decreases the energy release rate and toughens the composite. Moreover, when the heterogeneity contrast increases, the crack does not propagate continuously but jumps from an equilibrium position to another. This transition from continuous to intermittent crack propagation, often referred to as the "weak"-to-"strong pinning" transition (Patinet et al., PRL, 2013), generates additional toughening of the material, as the crack only visits the regions of high effective fracture energy.

Overall, we provide a comprehensive view of material toughening by heterogeneities for coplanar crack propagation, emphasizing on the role of geometrical nonlinearities and instabilities. Future works will focus on extending this framework to non-coplanar propagation, to account for the impact of surface/front roughening, and to cohesive failure, to capture size effects introduced by the presence of a finite process/damage zone.