Dynamics of axially traveling string with fluctuating speed and curved obstacles at both ends

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Abstract

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This research investigates the transverse vibrations of an axially traveling string against curved obstacles at both ends. The axial velocity of the string is fluctuating, which makes the system parametrically excited and leads to fairly complex dynamics. Floquet theory is employed to conduct the linear stability analysis. Stability diagrams illustrate the regions in the operating parameter space where the straight configuration and the associated vibrations are stable/unstable. We also investigate the influence of the resulting non-uniform tension on the stability regions.

Introduction

Several engineering systems like band saw blades, threadlines, high-speed magnetic tapes, and pipes transporting fluids involve the transverse vibration of an axially moving continua. More specifically, our system is pertinent to applications like ropeways and conveyor belts, which have big pulleys at the ends. Due to their technological importance, various researchers have explored the dynamics of axially moving systems. Wickert and Mote (1,2) studied the free and forced linear dynamics of axially moving continua by employing an eigenfunction method specifically for discrete gyroscopic systems or the Green's function method to obtain the dynamic response of axially moving string subjected to a traveling mass. Pakdemirli and Ulsoy (3) used the method of multiple scales to get the stability boundaries for an axially accelerating string and found that instabilities were observed when the frequency of the velocity fluctuations approached twice the natural frequency of the system at the mean velocity. More detailed studies on axially traveling systems can be found in (4-6). However, these studies focused solely on the dynamics of axially traveling strings, with no obstacle present at the ends as well as ignored any axial variation in the tension due to velocity fluctuations. The incorporation of non-uniform tension introduces linear coupling, modifying the stability boundaries. Furthermore, interaction with the curved boundaries introduces complexity resulting in nonlinear effects and more intricate dynamic responses that have been incorporated into our study. A comprehensive linear stability analysis that incorporates both uniform and non-uniform tension offers new insights into the dynamic behaviour of the system.

The current study assumes that the string travels with harmonically fluctuating velocity about some mean velocity. The primary features of these systems consist of velocity-dependent natural frequencies and the existence of a critical speed at which divergence instability arises. The wrapping and unwrapping of the string around the obstacle introduces

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a moving boundary (7), which, with a suitable scaling, is converted into a fixed boundary problem at the expense of introducing nonlinearities in the governing equation of motion. The variational principle has been used to derive the governing equation of motion and the appropriate boundary conditions. This system comes under the category of parametrically excited systems because of the presence of time-dependent terms in the governing equations. Previous studies (1-6) used either the stationary or traveling modes to discretize the governing partial differential equation. Stationary modes have slow convergence, whereas projections of nonlinear terms with traveling modes in our study are quite tedious. Hence, we use the assumed mode method to get the reduced-order system of ordinary differential equations governing the transverse vibrations of our system. In this method, the Lagrangian of the system is obtained in terms of the assumed solution for the string displacement. Then, the application of the Euler-Lagrange equation provides the system of ordinary differential equations that govern the dynamics of our system. We validate our model by comparing the linear vibration characteristics against those obtained from the discretized system using the stationary and traveling modes.

This validated model is used for linear stability analysis wherein we linearize our system about the static configuration of the string and study the linearized system using Floquet theory. The absolute values of the eigenvalues of the monodromy matrix, termed Floquet multipliers, depict the stability of the system. If any floquet multiplier is greater than one, then the system is unstable; if it is less than one, the system is stable. We plot the stability diagram between the forcing frequency and the fluctuating velocity amplitude for a fixed mean velocity value. We examine two cases, one in which tension remains uniform and the other where the tension varies spatially. In realistic cases, the tension typically changes due to the acceleration of the string, making it essential to consider non-uniform tension for a more accurate analysis.

Results and discussion

It is concluded that the dependence of the natural frequency on the traveling velocity is countered by the presence of the curved obstacles. It is observed that the discretization of the partial differential equation using the stationary modes shows slow convergence and involves some spurious roots. Discretization using the traveling modes is exact but complex. The assumed mode method is simple and gives fewer spurious roots than discretization using stationary modes. Linear stability analysis illustrates the influence of mean velocity, fluctuating velocity amplitude, and excitation frequency on the stability of the system. The spatial variation of tension influences the stability regimes. In contrast to the constant tension example, it enlarges the instability regime for lower mean velocity values while decreasing the number of unstable points in the parameter space at higher mean velocities.

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