
Continuum mechanics of non-affine fibre network materials

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Abstract

Random fibre networks characterise the microstructure of various natural and man-made materials, endowing them with fascinating deformation and fracture properties at the macroscopic length scales (1-3). These special characteristics are strongly linked to the microkinematic behaviour at the fibre-scale, which is typically non-affine, i.e. the fibres deform differently from vectorial line elements in a continuum. This non-affine relation between macroscopic and fibre-scale deformations still leaves a gap in the development of microstructurally motivated constitutive models for this class of materials, since even if the single fibre's non-linear material properties are known, their state of deformation cannot be inferred from the macroscopic one.

Computational models of discretized fibre networks certainly circumvent this problem and have become the method of choice to gain understanding of the intricate deformation mechanisms of the network, to study the microscopic origin of special macroscopic characteristics in representative volume elements, or to even analyse reasonably sized problems by a full fibre-discretisation of the domain of interest. What is more, the analysis of the corresponding simulations can also be used to inform new continuum mechanical approaches to model the homogenized behaviour of these materials which overcome the limitation of affinity and hence close the constitutive gap. Based on such analyses, we have recently shown that the distribution of stretch within the network represents a suitable target for constitutive modelling of both affine and non-affine central force networks of elastic fibres (4,5). It makes the latter case amenable to continuum mechanics principles, and brings numerical benefits over traditional approaches in the former (6,7). Essentially, the deformation-dependent stretch distribution within a fibre network is a central characteristic property, which maintains information about the micromechanical state on the one hand, and serves to compute the homogenised macroscale mechanical response on the other hand.

In the present contribution, the theory will be reviewed, and an extension will be presented to account for fibre failure, and hence damage and finally fracture at network and homogenised scales, respectively. To this end, we analyse the stretch distributions in computational models of brittle fibre networks, and prescribe their evolution in terms of macroscopic strain,

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representing the homogenised state of network deformation. In addition to informing a continuum mechanical model suitable to simulate the fracture behaviour of fibre network materials, these studies reveal how effectively loads are redistributed within random networks after fibre failure, and hence shed light on the outstanding defect tolerance of these materials (cf. 3).

References

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