Stop Motion Computation for Masonry Arches according to Limit Analysis Theorem

Claudia Cennamo*†[1], Luciana Di Gennaro[1], Luigi Massaro[1], and Giorgio Frunzio[1]

[1]Dipartimento di Architettura e Disegno Industriale, Università degli studi della Campania "Luigi Vanvitelli" Via S. Lorenzo, 31, 81031 Aversa CE – Italy

Abstract

Around the mid-19th century, Jean Claude De Saint-Venant developed the theory of elasticity, which remained essentially a theoretical study until, at the end of the same century, François Hennebique presented the scientific community with a new challenge: reinforced concrete, a new material to which De Saint-Venant's theories were well suited.
Following the enthusiasm generated by the discovery and subsequent use of reinforced concrete in construction, the elastic theory served as the foundation for any computational calculations for over a century.

At the end of the 20th century, thanks to some brilliant Italian professors such as Edoardo Benvenuto (1), Salvatore Di Pasquale (2), Antonino Giuffrè (3), et al., as well as international scholars like Jacques Heyman (4), who embarked on a research path that moved beyond the now-classic approach embodied in elasticity theory, the scientific community came to the realisation that the built heritage, the historical patrimony of which all of Europe is still rich, could not be computed using the same theoretical models applied to reinforced concrete or steel.

The scholars, therefore, resumed reading ancient treatises and began to question how historical monuments managed to maintain such good health after millennia, having been built in an era when modern scientific theories were unknown.

Thus, the scientific community (5 - 10) began to seek alternatives to the classic duality between force-displacement and stress-strain systems, which were not always effective, especially when dealing with materials that had characteristics such as lack of tensile strength, sudden and brittle failure, and absolute incompressibility.

Masonry, which constitutes the entire architectural heritage, generally exhibits all these characteristics, but it also has multiple typological aspects, making it an oxymoron, indefinable by definition, which scholars can only manage by grouping its most salient and common features, with the understanding that each structural component must be considered individually.

In line with the above, the present work, based on Heyman's hypothesis of a rigid block, in the sense of a material that does not respond to tension, has infinite friction, and infinite

*Speaker
†Corresponding author: claudia.cennamo@unicampania.it

compressive strength, proposes a model of a rigid-labile arch structure that is not subject to problems of strength but only of geometry.

The opening of four unilateral hinges is in predetermined positions to enable the activation of a kinematic mechanism. The position of the hinges is obtained by identifying the smallest collapse multiplier that is kinematically admissible (11).

It is possible to apply the Principle of Virtual Work to calculate displacements at a given time t_i, being the kinematic chain determined. The superposition of the time steps for i=1 to n determines the static collapse of the structure resulting from the alignment of the hinges.

The model provides a simple approach derived from traditional frame-by-frame cinematic animation. Allowing each successive configuration to be identified by iterating the previous one enables the frames in succession to create movement, with the advantage of obtaining a dynamic overview and an easier computation that does not rely on step-by-step analysis.

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