Efficient Wavelet-FFT-Approaches for Multiscale Problems

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Abstract

The predictive capability of computational multiscale methods relies on detailed representations of the underlying microstructure and on sophisticated material models for the microscale constituents. However, with the increasing physical complexity of the cell problem, the computational complexity also keeps increasing. This necessitates the development of tailored solution approaches that leverage the specific structure of the cell problem, with FFT-based spectral methods emerging as some of the most promising options.

In this work, we address a key drawback of state-of-the-art FFT-based solution approaches: their reliance on regular structured grids. To overcome this, we make use of the intrinsic hierarchy and built-in adaptivity of wavelets. By representing the governing fields in a wavelet basis and by using wavelet transforms, higher-order stress approximations are successively derived within a nested set of approximation spaces. This enables the detection and accurate resolution of localized features while (at the same time) significantly reducing the number of material model evaluations. Since the computational overhead of the wavelet transform scales linearly in the number of voxels, and since the additional per-voxel operations are negligible compared to the cost of typical material model evaluations, a substantial gain in computational efficiency is achieved.

This is demonstrated through a detailed study of representative boundary value problems in one- and two-dimensional settings, where we use a wavelet-enhanced version of the classic Moulinec-Suquet basic scheme as a starting point. In particular, we show that the numerical grid in the hybrid wavelet-FFT approach naturally adapts to the solution profile based on a predefined refinement tolerance and report a 95%-reduction in the number of material model evaluations.

(1) T. Kaiser, T. Raasch, J.J.C. Remmers and M.G.D. Geers: *A wavelet-enhanced adaptive hierarchical FFT-based approach for the efficient solution of microscale boundary value problems,* Computer Methods in Applied Mechanics and Engineering, 409, 115959, 2023, doi: 10.1016/j.cma.2023.115959

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