# Mathematical and numerical analysis of Data-Driven Identification (DDI) method: the case of isotropic materials

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## Abstract

Identifying the mechanical responses of materials is a fundamental challenge in solid mechanics. The constitutive equations close the boundary value problems of mechanics by establishing a relationship between kinematics (strain) and equilibrium (stress). This relationship can be identified through mechanical tests: simple tests (homogeneous isostatic tests) or complex tests. Simple tests on the material allow for direct calculation of the stress field from a force measurement. These tests are limited to simple deformation modes and often require multiple tests to obtain sufficient data for identifying the stress field. In contrast, complex tests involving heterogeneous strain fields provide more information in a single test, although the stress field cannot be determined directly using traditional methods.
Classically, the stress field is estimated using identification methods such as Finite Element Method Updating (FEMU), Virtual Field Method (VFM), among others. These methods rely on a postulated model a priori with its parameters adjusted to fit experimental data. However, the initial choice of the predefined model strongly influences the results and adds bias.

An alternative identification approach called Data-Driven Identification (DDI) was proposed by Leygue *et al.* (2018), allowing the estimation of stress fields from full-field kinematic measurements and applied net force without postulating a constitutive model. This method is based on Data-Driven Computational Mechanics (DDCM) introduced by Kirchdoerfer and Ortiz (2016), where a database of admissible strain-stress pairs samples the mechanical response of the material. DDI builds this database, which is used to regularize the stress estimation problem, but generally admits multiple solutions.

DDI has been successfully applied to synthetic and real data for various material behaviors, including elasticity, hyperelasticity, viscoelasticity, elastoplasticity, and large deformation. Although, a theoretical study of the method to validate the ability of DDI to accurately estimate and converge to the true mechanical stress is still missing.
In this work, we investigate numerically and algorithmically the preliminary work of Leygue (2024), where a criterion for the uniqueness of solutions was introduced. We propose three methods to characterize this criterion numerically and will evaluate their effectiveness, including a comparison of their computational costs. The DDI framework relies on algorithmic

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parameters that are often chosen empirically. As these parameters are adjusted to gain accuracy, the DDI becomes ill-posed. This study will help us select DDI parameters, reducing the risk of over-fitting. We extend the algebraic framework presented in Leygue (2024) to demonstrate how one can drive DDI to account for the possible isotropy of the materials. The consequences of this isotropic assumption are discussed.