A Model for the Dynamic Snap-Through of an Elastica

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Abstract

We study the problem of dynamic snap-through of a pre-buckled arch. Predicting the occurrence of the instability is straightforward, as is identifying the equilibrium configuration to which the arch jumps from a limit point. However, less is understood about the dynamic nature of the transition itself. We propose a new model to describe the dynamics of the arch during the snap-through. The model is validated with fully dynamic simulations and experimental measurements and provides insights that are immediately useful in designing robotic devices that function by exploiting snap-through instabilities for locomotion.
Specifically, we study the symmetric snap-through of a pre-buckled arch with clamped ends. The elastic response of the arch is characterized by the ratio d/L, with d denoting the distance between the clamps and L, the length of the arch. When the clamps are rotated to a critical angle that depends on d/L, the arch snaps spontaneously and dynamically to a new equilibrium configuration. The instability is displacement/angle controlled, and in particular, not load-controlled. The snap is typically visualized on a plot of equilibrium solutions. Such a depiction is convenient but misleading since the arch is never at equilibrium during the transition. In reality, the arch snaps past the stable equilibrium and vibrates about this configuration. Recent studies have focused on the arch's dynamic behavior near the unstable equilibrium point (critical slowing down). Instead, our work identifies an approximate, albeit useful, description of the dynamic evolution of the arch during the snap-through event.

To study the instability, we devise benchtop experiments to initiate it, record forces and moments, and image its configurations while ensuring that the arch profile remains symmetric. For a spring-steel arch with L_˜25 cm and d/L_˜0.95, the arch snaps within about 80 ms, and its midpoint reaches a peak velocity of about 6m/s. These observations reinforce the snap-through's dynamic nature and inertia's significance. This stands in contrast to the quasi-static nature of buckling instabilities. Over an extensive range of d/L ratios, examining the sequences of profiles of the arch during its snap-through reveals that its midpoint's displacement dictates the arch's configurations. Specifically, we find that the arch profiles during its dynamic transition are indistinguishable from strain energy-minimizing configurations with identical boundary conditions at the clamps and with coincident midpoint displacements.
Based on this observation, we propose a model that identifies the sequence of arch configurations during its dynamic transition as a 1-parameter family of strain energy-minimizing configurations, with the parameter being the midpoint displacement. Then, imposing conservation of energy yields a time-dependent sequence of configurations. From a computational perspective, the model only requires the resolution of ODEs. We validate the model's efficacy

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through comparisons with fully dynamic simulations and experiments over a wide range of d/L ratios that far exceed the expected range of use in robotic applications. The model's reliance on strain energy minimization, and consequently on quasistatic solutions to determine dynamic configurations, may appear paradoxical. However, this is not the case. In fact, the model can be interpreted as identifying the "most dynamic" transition between the unstable and stable equilibria or as one that minimizes the midpoint's time of travel rather than the Hamiltonian. In the presentation, we will discuss these aspects of the model and its application to designing underwater robots that locomote by harnessing the energy released during the snap-through.