Constitutive modeling of hyperelastic materials based on the upper triangular decomposition

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Abstract

The upper triangular decomposition provides an alternative method to multiplicatively decompose the deformation gradient tensor into a product of a rotation tensor and an upper triangular distortion tensor (1,2). The six components of the distortion tensor can be directly related to pure stretch and simple shear deformations, which are physically measurable. In this study, constitutive equations for hyperelastic materials are derived using strain energy density functions in terms of the distortion tensor. An untangled crosslinked network model and a tube-like constraint model are employed along with inverse Langevin function approximations to construct four strain energy density functions. The Cauchy stress components, derived directly as partial derivatives of the strain energy density function with respect to the distortion tensor components, have simpler expressions than those based on the invariants of the right Cauchy-Green deformation tensor. To validate the newly proposed constitutive equations, four fundamental deformation modes - uniaxial tension, pure shear, equi-biaxial tension, and simple shear - are considered. Experimental data from rubber and brain tissues are utilized to calibrate the model parameters. The fitting results reveal that each of the four proposed constitutive models can effectively capture the mechanical behaviors of hyperelastic materials using a single set of parameters. Hence, these models furnish a practical alternative approach to describing hyperelatic materials based on the upper triangular decomposition of the deformation gradient tensor, offering measurable and physically interpretable constitutive relations.

References

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