
Buckling of a hanging elastic sheet

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Abstract

This talk will explore the buckling behavior of a square, intrinsically flat sheet suspended from one of its corners. If the sheet is sufficiently stiff, it will resist deformation due to gravity and remain flat. On the other hand, if the sheet is sufficiently compliant, it will buckle under its own weight into a wrinkled configuration, with each successive buckling mode introducing an additional wrinkle-similar to the buckling modes of an elastic column. However, unlike a column, we show that higher-order buckling modes can become stable for sheets that are sufficiently large, heavy, or compliant. We approximate the sheet's deformation by first assuming that the sheet is developable and then that every generator spanning the developable surface intersects the corner from which the sheet is suspended. Under these assumptions, the problem of finding stable equilibrium configurations of the two-dimensional sheet can be reduced to a one-dimensional calculus of variations problem. We then show via an application of Noether's theorem that the Euler-Lagrange equations for this problem can be integrated to find a closed-form expression for the sheet's curvature. These results are used to compute the critical combinations of geometry, weight, and stiffness at which the flat sheet buckles into a wrinkled configuration. Finally, we apply numerical continuation methods to compute the sheet's post-buckled configurations and determine their stability. We find that some higher-order solution branches undergo pitchfork bifurcations, through which wrinkled configurations become stable.

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