Three-dimensional limit analysis through the Thrust Surface Method to investigate the structural behaviour of Apulian star vaults

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Abstract

The investigation of the structural behaviour of masonry vaults remains a subject of ongoing research. Indeed, conventional structural analysis approaches reveal to be inadequate for understanding the complex mechanical behaviour of these constructions, governed by anisotropy, material non-linearity, damage, plasticity and fracture. The employ of advanced computational models is obstructed by masonry's intrinsic irregularity and heterogeneity, which clash with the in-depth mechanical characterization and accurate definition of boundary conditions required.
These difficulties suggest to shift away from searching for the full characterization of the masonry structural response, and instead focus on estimating collapse loads following the path traced by Heyman's studies (1), which reread ancient geometrical theories into the modern framework of Limit Analysis, highlighting that the safety of a masonry structure depends substantially on its geometry.

For a fully three-dimensional Limit Analysis masonry vaults and domes, several approaches have been recently proposed. In particular, some of the Authors have recently developed the Thrust Surface Method (TSM) (2), a new computational method framing the equilibrium problem into a constrained optimization problem. This way, "extremal" or "optimal" statically admissible solutions are obtained, capable of fully exploring the entire load-bearing capacity spectrum of a vault or a dome having an arbitrary geometry, also considering horizontal loads such as those representing maximum inertia effects from seismic actions. Through a convenient numerical formulation, no a priori assumptions are required, making it possible to simultaneously determine the two unknown functions of the problem: the Airy stress function **F** and the membrane shape function *f*. TSM could be a valuable tool for visualizing and understanding the complex three-dimensional behaviour of masonry vaults, highlighting the intricate relationship between form and structure.

Here we apply TSM for deepening the structural behaviour of star vaults, a typical vaulting system that characterizes mainly the historical Apulian architecture (southern Italy). The complex shape of the star vault originates from the intersection of a cross vault with an ellipsoidal dome: this makes the structural analysis quite challenging. Indeed, despite their widespread, star vaults have been insufficiently investigated and their mechanical behaviour

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is not completely understood yet.

With the use of TSM, the capability of analysing optimal solutions provides a comprehensive assessment of the load-bearing capacity spectrum of the star vault. Indeed, the investigation of the vault's complex three-dimensional behaviour reveals a strong spatial distribution of stresses on the central dome area, which cannot be accurately represented by flat patterns, highlighting how traditional approaches may be overly conservative.

Furthermore, evaluations of critical stress states near the folding lines characterizing the vault's geometry are made.

Finally, the substantial differences with the well-known cross vault are studied, and the corresponding load-bearing capacity is also compared.

This yields critical insights into the relative safety levels of the two vault typologies for the same covered span.

References

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