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# Prediction of local field statistics using an incremental mean-field homogenization scheme for high-concentration composites

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## Abstract

To predict the non-linear behavior of composites using mean-field homogenization methods, it is necessary to estimate local field statistics (1). In the case of composites with a high concentration of inclusions, the differential scheme in its incremental form has already proved its worth. Our aim is to define local field statistics within the framework of this incremental method.

General expressions of the first and second-order moments of the intraphase strain fields have been derived for multiphase elastic composites. The main challenge in this work was to account for the successive homogenization steps inherent to the incremental process. For first-order moments, the localization tensors for each phase were achieved by reconstruction by integrating all the homogenization steps. For second-order moments, the approach is based on the application of the chain derivation rule throughout the process, making it possible to express the derivatives of the final effective properties as a function of the initial properties of the phases.

On the one hand, this approach has been validated by comparison with Zimmerman’s analytical solution for isotropic porous media (2). On the other hand, additional applications were carried out on microstructures presenting phases with pronounced contrasts and a wide range of volume fractions. These results were then compared with those of other linear homogenization schemes, such as the Mori-Tanaka (3) and Lielens schemes (4), as well as with full-field simulations. These comparisons confirm the relevance of this scheme in the context of highly inclusionary media.

(1) P. Ponte Castañeda and P. Suquet, "Nonlinear composites," *Adv. Appl.Mech.*, vol. 34, pp. 171–302, 1998.

(2) R. W. Zimmerman, "Elastic moduli of a solid containing spherical inclusions," *Mechanics*

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*of Materials*, vol. 12, no. 1, pp. 17–24, 1991.

(3) Y. Benveniste, "A new approach to the application of Mori-Tanaka's theory in composite materials," *Mechanics of Materials*, vol. 6, no. 2, pp. 147–157, Jun 1987.

(4) G. Lielens, P. Pirotte, A. Couniot, F. Dupret, and R. Keunings, "Prediction of thermo-mechanical properties for compression moulded composites," *Composites Part A: Applied Science and Manufacturing*, vol. 29, no. 1-2, pp.63–70, 1998.