# Foundational aspects of an hybrid equilibrium element for finite poroelasticity

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**Abstract**

Computational modeling of coupled diffusion problems in the finite strain regime represents an area of considerable complexity arising from the intricate interplay between the deformation of a porous solid matrix and the fluid flow permeating the interconnected pore network. A significant amount of literature covers a wide range of finite element formulations, most of which are based on two-field discretization – typically incorporating solid displacements and pore pressures – or a more complex three-field discretization incorporating solid displacements, which commonly remain a primary variable (1, 2).
From a variational perspective, these methodologies lead to saddle point problems that require the implementation of stabilization techniques to ensure accurate and reliable solutions. A basic requirement to infer the well-posedness of these formulations is adherence with the inf-sup condition (3). However, even if this criterion is met, substantial challenges can arise in the numerical solution, especially in low permeability or small time step scenarios. In such cases, the use of stabilization strategies or highly refined meshes is essential to mitigate instabilities such as spurious pressure oscillations, which can significantly affect the accuracy of the solution even for seemingly simple benchmark problems.

An alternative paradigm (4, 5) involves adopting a specific set of state variables that inherently preserves mass balance at the local level and allows the derivation of a variational minimization principle. By satisfying the inf-sup condition a priori, the latter approach yields an element with superior intrinsic stability. Nevertheless, the development of a conforming and computationally efficient element capable of locally enforcing the mass balance depends on the adoption of an appropriately reduced numerical integration scheme for the volumetric coupling term. Recently, a related stress-pressure formulation (6) in the small deformation regime has led to the development of a hybrid equilibrium element that does not rely on stabilization techniques to cope with spurious pressure oscillations.

Building on these advances, the present work presents a novel hybrid equilibrium finite element formulation to address the nonlinear coupled problem of fluid transport in porous media. By exploiting the intrinsic properties of hybrid elements, the proposed approach facilitates the enforcement of fundamental balance laws within the discretized framework.

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This property not only increases the theoretical rigor of the formulation but also avoids numerical artifacts, thereby eliminating the dependence on external stabilization techniques and providing a robust tool for accurate computational modeling of complex poroelastic systems. The related finite element implementation details are shown and finally the obtained numerical results for a few basic poroelastic problems are discussed.

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