# Implicit Non-Smooth Material Point Method for Non-Associated Elasto-plastic Soils

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**Abstract**

From footing simulations to slope stability analyses, geomaterials are frequently described by non-associated elasto-plastic constitutive models. To ensure dissipativity and exact resolution of the constitutive model at each time step, an implicit formulation of the numerical method is often preferred. It is typically combined with the classical return mapping algorithm (Simo et al., 1989). This work, in continuity with Acary et al., (2023), proposes an alternative approach through the development of a monolithic solver capable of simultaneously solving the plastic constitutive equation, the potential hardening and the Coulomb friction with unilateral contact. This approach avoids in particular the challenging computation of the consistent tangent operator and enables the use of efficient and robust solvers that come from numerical optimization.

In order to account for case studies where, as it is the case with slope failures, the material is subjected to a loading phase followed by a flow phase exhibiting large deformations, we use the Material Point Method (MPM). The method, developed by Sulsky et al. (1994), employs both Eulerian and Lagrangian discretisation, and thus making it well suited for this class of problems. To allow for a rigourous comparison with the robust and frequently used Finite Element Method (FEM), the solver is first developed in an FEM framework then adapted to MPM. The constitutive equations are written as a differential inclusion, inspired by the Implicit Standard Material (ISM) framework (Berga and De Saxcé, 1994). A semi-smooth Newton method (Alart and Curnier, 1991; Christensen, 2002; Hager and Wohlmuth, 2009) and a non-smooth Gauss-Siedel method (Jourdan et al., 1998) are used to solve the constitutive law within an implicit time scheme.

Among its favorable mathematical properties, we highlight the good energy balance of the solver and the existence of solution in non-associated case. The convergence can be proven analytically in associated cases while in practice the solver shows in any cases quadratic convergence, even for large time steps, or load-increments. The monolithic structure of the

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solver permits the implementation of non-conforming boundary conditions inspired by contact dynamics. The relevance of the method is demonstrated with a footing problem and slope stability simulations where the existence of an analytical solution in limit analysis is established (Chen, 2013). The simulation demonstrates a high degree of correlation between the MPM and the analytical solution with regard to ultimate bearing capacity, as well as a good adequation between the MPM and the FEM stress and strain distributions. Further simulations are performed to account for the transition from load to flow in the case of slope instability.

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