Nontrivial and Nonlinear Behavior of Negative Gauss Curvature Shells

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Abstract

Thin elastic shells, defined by their inherent curvature, are commonly found in nature and engineering due to their remarkable structural efficiency. While the mechanics of shells with zero (e.g., cylindrical and conical) and positive (e.g., spherical and ellipsoidal) Gaussian curvature has been widely studied (1), the nonlinear response of shells with negative Gaussian curvature has been far less explored. Among these thin elastic shells, the positively curved shells, such as pressurized spheres and ellipsoids, are particularly known for their "geometry-induced rigidity," where local curvature significantly enhances resistance to deformation (2). In contrast, negatively curved shells, including saddle-shaped surfaces, are expected to display markedly different nonlinear responses.

In this talk, we will report results from a combined experimental and numerical investigation of shell structures constructed from triply periodic minimal surfaces (TPMS), such as the Gyroid discovered by Alan Schoen in 1968 (3). These surfaces, characterized by zero mean curvature and non-positive Gaussian curvature, can serve as models for more complex, interconnected geometries found in natural and engineered systems, from cellular structures to metamaterials (4). We will present experimental results from mechanical tests of representative examples of these negative curvature shell structures. We seek to characterize and rationalize the nonlinear mechanical response of TPMS and other intricate shell-like geometries. Experimentally, we employ rapid prototyping techniques, focusing on 3D printing, to fabricate thin elastic shells with controlled geometric and material properties. We then perform mechanical tests on these samples, subject to various loading conditions, including indentation and pressurization, to assess their stiffness, buckling thresholds, and post-buckling behavior. Moreover, the sensitivity of negatively curved shells to imperfections is explored systematically by numerical simulations using the finite element method.

The findings of this research yield fundamental insight into the underlying deformation mechanisms and the role of geometric nonlinearities in the mechanical properties of TPMS and other complex geometries. Our work on the mechanics of negatively curved shells provides a predictive understanding of the rational design of lightweight, shape-morphing, and energy-absorbent structures and metamaterials.

References

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