Spin-It Faster: Quadrics Solve All Topology Optimization Problems That Depend Only On Mass Moments

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Abstract

The behavior of a rigid body primarily depends on its mass moments, which consist of the mass, center of mass, and moments of inertia. It is possible to manipulate these quantities without altering the geometric appearance of an object by introducing cavities in its interior. Algorithms that find cavities of suitable shapes and sizes have enabled the computational design of spinning tops, yo-yos, wheels, buoys, and statically balanced objects. Previous work is based, for example, on topology optimization on voxel grids, which introduces a large number of optimization variables and box constraints, or offset surface computation, which cannot guarantee that solutions to a feasible problem will always be found.

In this work, we provide a mathematical analysis of constrained topology optimization problems that depend only on mass moments. This class of problems covers, among others, all applications mentioned above. Our main result is to show that no matter the outer shape of the rigid body to be optimized or the optimization objective and constraints considered, the optimal solution always features a quadric-shaped interface between material and cavities. This proves that optimal interfaces are always ellipsoids, hyperboloids, paraboloids, or one of a few degenerate cases, such as planes.

This insight lets us replace a difficult topology optimization problem with a provably equivalent non-linear equation system in a small number (< 10) of variables, which represent the coefficients of the quadric. This system can be solved in a few seconds for most examples, provides insights into the geometric structure of many specific applications, and lets us describe their solution properties. Finally, our method integrates seamlessly into modern fabrication workflows because our solutions are analytical surfaces that are native to the CAD domain.

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