Local Impenetrability in Slender Bodies of Finite Thickness

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Abstract

A slender body is a solid whose one dimension is much larger than the other two. Theoretical modelling of a slender body generally entails reducing it to a centerline curve, and augmenting it with a frame of directors that track the flexure of the body in space.
The lateral dimensions, i.e. the cross-sectional thicknesses, of the slender body do not explicitly feature in such theories, except by way of a bending modulus, that penalises flexural deformations of the curve.
In this talk, we will discuss the effect of the thickness of a slender body via the local impenetrability of the material surrounding the centerline curve.

By local impenetrability of the surrounding material, we mean that the determinant of the deformation map, connecting the tangent spaces of the reference and deformed configurations, must remain positive-definite. We compute the consequences of this constraint on the kinematics of a slender body of circular cross-section, and derive an inequality constraint connecting the Frenet-curvature, the centreline stretch, and the thickness of the body. For inextensible deformations, the relation reduces to an upper-bound on the Frenet-curvature of the centerline.
We incorporate this inequality constraint within the theory of Kirchhoff elastic rods using a variational principle. We derive the corresponding Euler-Lagrange equations for our system, and note that local impenetrability modifies the constitutive relation between the internal moment and the curvatures of the centerline. As an illustrative example, we use our theory to compute the shape of a fully flexible string hanging under gravity, and show that below a critical horizontal distance between the two ends of the string, our solutions deviate from the well-known classical catenary solution. Finally, we validate our results with some table-top experiments, and discuss the implications of our theory on the computations of ideal shapes of clasps and knots.

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