
Non-material finite element model for the contact problem of sliding extensible rods

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Abstract

We present a numerical investigation into the statics of two flexible rods, one partially inserted into the other, thus considered as a flexible sleeve. Both rods are subjected to external moments at their simply supported ends. The insertion naturally divides the system into three segments with the overlap as the central one. Here, the centerlines of the rods coincide while allowing for frictionless sliding. The variability of the overlap's length leads to configurational forces being exchanged between the rods in addition to concentrated moments and a distributed force.

Although recent theoretical analysis has provided foundational insights towards finding the specific values of the configurational forces, computational validation of these results remains absent. Besides, previous numerical studies focused on the case of inextensible centerlines, precluding the determination of the normal force distribution among the rods within the overlap. This work advances the field by incorporating an elastic constitutive relation that links the normal force to the axial strain, resolving the static indeterminacy and enabling the computation of these quantities throughout the entire system.

However, extensibility introduces significant complexity to the problem, particularly as the material lengths of the rods within the overlap generally differ. Consequently, the system requires two configurational parameters instead of one to describe the segment partitioning uniquely. Moreover, finite element modeling is challenging in this context. A purely Lagrangian formulation is cumbersome due to non-coincident nodes of the two rods in the overlapping region, and a purely Eulerian approach is unsuitable due to the spatial mobility of the transition points between segments.

To address these challenges, we propose an approach characterized by the following properties: The approximated quantities are the transverse deflection y and the material coordinate s , represented as piecewise functions of an auxiliary parameter σ , which ranges from 0 to 1 in each segment. The mapping between σ and the spatial axial coordinate x is chosen in a piecewise linear fashion. Within each segment the nodes $\sigma=\text{const}$ are distributed equidistantly with respect to σ and hence also x . Cubic Hermite polynomials are employed to interpolate y and s , ensuring C1-continuity within segments, while additional procedures establish the necessary degree of continuity at transition points. Providing all the boundary, transition and overlap conditions, we apply the principle of virtual work for the conservative system at hand and finally solve for equilibrium at static or kinematic loading.

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The computational results are compared with theoretical predictions. From one side, we can evaluate the curvature and thus the bending moment in the transition points. Similarly, the axial forces at the rods' central ends can be obtained via the longitudinal strain. As a simple consequence of the boundary conditions for the individual rods, these forces are exactly equal to the configurational ones. Conducting these calculations, it is possible to exemplarily verify the general relation between bending moment and configurational force obtained from previous analytic investigations.

Again, using the constitutive law for extensibility, the axial force distribution within the overlapping region can be computed. Not only does this provide a novel insight, but it also allows for further comparisons with analytic results. In case of inextensibility, the rods are known to each have an invariant – the Hamiltonian, which is conserved even transitioning from one segment to another. For extensible rods with finite yet high tension stiffness relative to bending stiffness, we hypothesize that the Hamiltonian retains its role as a quasi-invariant. Indeed, numerics reveal no significant change in this quantity, especially when passing the segments' borders.

In conclusion, this study provides a computational validation of recent theoretical findings, including the relationship between bending moment and configurational force for a flexible rod, sliding in a flexible sleeve. It also sheds light on axial force distribution within the overlap and demonstrates the quasi-invariant behavior of the Hamiltonian under high tension stiffness. These contributions not only confirm analytic results and deepen our understanding of mechanical systems featuring sliding contact of elastic structures, but also provide a further example of mixed Eulerian-Lagrangian kinematic description for contact problems.