Symmetries of periodic and quasiperiodic heterogeneous materials: From invariance to indistinguishability

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Abstract

The effective behavior of heterogeneous materials is governed by the spatial arrangement of the matter on a microscopic scale, in other words by its microstructure.
Microstructures are classically divided into two families, periodic ones and random ones. Each has its own strengths and weaknesses, as well as its own techniques for calculating the resulting effective physical properties. Periodic microstructures are deterministic but often anisotropic, while random microstructures have higher toughness at the cost of high variability between samples. In recent years, these classical boundaries have been blurred since new paradigms of spatial organization have emerged from the introduction of long-range order into random media (hyperuniformity) (1) and local disorder into periodic ones (quasiperiodicity).

The presented works focus on the latter option of quasiperiodic order. These materials are appealing because they allow higher-order rotational symmetries while losing translational invariance. The result is a highly isotropic, deterministic structure with high resistance to crack propagation (2). However, the classic techniques and concepts of periodic homogenization need to be completely revised.

Knowing material spatial symmetries enables us to determine, via Currie's principle, the symmetry class of the effective property (3). This information is useful either for validating a traditional numerical homogenization model or for predicting the number of effective material parameters in a data-driven approach.
However, for quasiperiodic materials, the classical definition of symmetry is meaningless, since the strict criterion of invariance can no longer be satisfied. Hence the first point to revisit is the notion of spatial symmetries.

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This generalization amounts to substituting the more general concept of indistinguishability for the concept of invariance. Two media are said to be indistinguishable if their n-point correlation functions are equal at any order n. This criterion, formulated in Fourier space, takes a particularly elegant form (4) which allows for direct numerical evaluation.

Following this path, we propose a numerical method operating on gray scale images that detects the symmetries of both periodic and quasiperiodic microstructures in Fourier space. The designed tool can be applied to less ordered structures, as it gives a meaningful definition of symmetries in an approximate sense.

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