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# On the computational methods for the analysis of transversely flexible sandwich beams

Tiago Balestro\*<sup>1</sup> and Adelano Esposito<sup>1</sup>

<sup>1</sup>Federal Institute of Education, Science and Technology of Rio Grande do Sul – Brazil

## Abstract

Sandwich structures are already well known in fields like aerospace due to their high stiffness-to-weight efficiency. However, by adding a flexible core, new interesting properties arise. In comparison with a single layer structure or a rigid sandwich, the configuration of flexible sandwich structures enables more complex deformation mechanisms, specifically, a transversely flexible core allows the face sheets to deform semi-independently, enhancing flexibility and allowing a tunable shape. When combined with magnetorheological materials, for instance, these structures can have important applications in fields like adaptive structures, soft robotics, and morphable wings.

The foundational high-order theory of transversely flexible sandwich beams, introduced by Frostig *et al.* (1992), provides a closed-form solution, enabling analysis of vertical normal stress, in-plane shear stress, and displacement fields. The last allows researchers to predict structural deformation in advance by solving the governing equations and updating the displacement field equations.

Even so, the solution of the original model governing equations is not straightforward. In particular, Swanson *et al.* (1999) expanded the topic further by introducing a work on implementing the theory. Moreover, further studies have emerged over the years, suggesting the solution in a different way, based on the Fourier series (Phan *et al.*, 2012), (Lashkari *et al.*, 2016).

The governing equations of the theory constitute a set of five coupled linear ordinary differential equations of fourth order. Its complete solution provides the shear stress, as well as the longitudinal and vertical displacements of both the upper and lower faces. Such solutions are fundamental to a complete analysis of the sandwich, creating a displacement field of the core and evaluating stresses on the structure.

In this context, the present work discusses the solutions procedure further and analyses alternative methods: direct numerical solution, nondimensional equation simplification and solution by means of commercial Differential Equations Solvers. The computational algorithms are employed using Mathematica software, allowing easy mathematical implementation. The study also seeks to produce accurate graphical representation of the core deformation, an aspect that has not been extensively explored in prior research, yet crucial for predicting beam behaviour.

It was observed that although Fourier analysis can be powerful, expressing certain load

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\*Speaker

conditions in Fourier terms can be challenging, restricting practical applications. The direct numeric approach, which is simpler to implement at first, faced challenges due to ill-conditioned systems resulting from coefficients of widely varying magnitudes. The plot of the theoretical core deformation was simple to implement in Mathematica and produced excellent graphical results of the nonlinear deformation pattern, a high-order effect.

In the current stage, some methods are still being tested. The performance of all will be compared to the original solutions, even though these require extra computation as well, as outlined in Swanson. Furthermore, once the ODE system is solved, the computational model will be validated through bending tests on real transversely flexible sandwich structures. Especially for flexible structures, where displacements are significant, comparing the real displacement field to that predicted by the theory is important. Finally, special care will be taken with the performance of the computational model, since it aims to be used alongside structural optimization routines.

With the proposed studies concluded, it is expected to find a computational model that is both robust and user-friendly, enabling the simulation of transversely flexible sandwich beams under diverse static load conditions.

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